The Nature of the Loss Ratio in Property-Casualty Insurance

Peter M. Ellis

Abstract: The aggregate loss ratio in the United States stock property-casualty insurance industry is represented with an autoregressive AR(3) model. It is demonstrated with a Monte Carlo simulation that the loss ratio is quite unstable. Annual growth has been strong for both premium volume and losses, but the loss ratio is found to fluctuate unsteadily as an AR(3) process. Also, there is a continuing upward drift to the loss ratio that cannot continue without bringing failed loss coverage to the industry.

INTRODUCTION

Insurers, regulators and the public have a clear and continuing interest in preserving the viability of the insurance enterprise. Insolvencies almost certainly subject policyholders to financial chaos. Regulators are required to protect insureds from this calamity. The insurers also clearly wish to have excellent financial health. It is therefore necessary for the insurance enterprise to experience either underwriting gains or minimal losses. However, the total loss and expense ratio in property-casualty insurance is often in excess of 1.0 (Best’s Aggregates and Averages, various years), indicating that underwriting activity has been carried out at a loss. This may be acceptable even if it continues for years because investment gains have generally been steady and consistent. They historically have been relied upon to more than offset any underwriting loss.

Fairly (1979) carried out an excellent analysis of underwriting gain or loss in property-casualty insurance. He invoked the capital asset pricing model...
model to determine necessary rates of return on underwriting activity. One conclusion of his research was that there has been a historically continuing excess profit margin on net premiums written beyond that determined by his pricing model. It is to be noted that these margins are negative, both on actual historical experience and from the capital asset pricing model. The clear expectation is that investment gains will be used to bring about positive profits and increases in the surplus level.

It will be seen here that the loss ratio has been generally increasing over the last 55 years. This trend cannot continue indefinitely without forcing insolvencies. The nature of the increase will be found to have a complicated autoregressive nature with a great deal of variability. The resultant destabilizing effects upon underwriting gain or loss would render annual investment gains as the only reliable component of industry profitability.

The return on underwriting is determined by the combined loss plus expense ratio. The expense ratio is underwriting expenses/net premiums written and the loss ratio is (loss plus loss adjustment expenses)/net premiums earned. The total loss plus expense ratio is an indicator of the profitability of underwriting activity. Best’s Aggregates and Averages (various years) shows that expense ratios have stabilized at between 20% and 30%. Loss ratios, however, are shown here to have a complex time series nature. They do not appear to have any single limit that is being approached. They may be naively observed to fluctuate with a set cyclical-ity, but it will be shown below that the actual nature is not one of classical periodicity. This unsteady nature may well be explained by the constant adjustment in premium rates that arises from the opposing pressures of competitive pricing and loss experience.

Harrington (1984) published a survey of notable research on the question of rate regulation on underwriting results. It was a very large study and the references contained therein provide an excellent point from which to research the entire issue. He concluded that it is generally true that loss ratios are not consistently greatly different in states with competitive rating and noncompetitive rating. Regulators will examine the loss ratio because it is so important in measuring the financial strength of an insurer. BarNiv and McDonald (1992) traced the history of research into insolvency prediction for the insurance industry. They presented an insolvency prediction model that used seven independent variables. The included predictor variables surfaced as significant through two-stage regression. Three of these variables incorporated the loss ratio. Hence, it seems correct to conclude that the loss ratio is indeed an important determinant of solvency.
Cummins, Harrington and Klein (1995) presented a logit model of insolvency prediction that used a ratio of loss reserves to premiums written. This ratio will be close to the loss ratio because actual losses would be expected to closely match loss amounts that are reserved. They used this logit study to conclude that the NAIC financial strength model has great weakness in its ability to predict insolvency. The NAIC model requires the determination of risk-based capital needs. It expressly uses both premium volume and loss reserves.

The loss ratio, or the ratio of loss reserves to premiums, has been used in the above studies because there is an obvious connection among profitability, premiums, and losses. The loss ratio has been widely accepted in the past and it will be studied here. Grace and Hotchkiss (1995) used the sum of the loss ratio and the expense ratio, the combined ratio, in a study of externalities in the profitability cycle. Haley (1993) used the same ratio to show that underwriting profitability is related to short-term interest rates. Fields and Venezian (1989) also used the combined ratio time series to show the effect of interest rates upon profitability. Smith (1989) demonstrated a relationship between the loss ratio and bond yields. Cummins and Outreville (1987) used the inverse of the loss ratio to present a study of underwriting cycles. These references indicate the traditional use of the loss ratio and the combined loss and expense ratio in analyses of industry profitability. These studies have used both individual firm loss ratios and the industry aggregate loss ratio. The loss ratio study done here is thereby a continuation of a number of past works.

It is true that fluctuations in the loss ratio might arise from changes in either the numerator or the denominator. Either way, however, the impact upon operating profitability and underwriting viability is the same. When the ratio increases, there are more loss dollars to be covered by each premium dollar. Because the loss ratio has this power to quickly and easily indicate the balance between premiums and losses, researchers such as those cited above have used it for years in various industry financial studies.

This study will examine the loss ratio over time for the 55 years from 1939 to 1993 for the aggregate United States stock property-casualty industry. The data was obtained from Best’s (several years). A finding that the loss ratio continues to drift upward would be alarming because insolvencies would inevitably result. A finding that there is a high level of unexplained variability in the loss ratio would indicate that future underwriting gains and losses might have a strongly random nature.
The Loss Ratio Time Series

Figure 1 presents the time series pattern of net premium volume in the industry. The graph shows that there has been steadily increasing premium volume since 1939. Figure 2 presents the time series of aggregate losses in the industry. It is clear here as well that there has been steady growth over time. Figure 3 is quite different, however. It is the time series of the aggregate loss ratio. The graph shows a generally increasing loss ratio over time. It will be shown here that the loss ratio has a persistent time series nature that is not suitably portrayed with a simple random walk. The observed pattern in Figure 3 notwithstanding, the loss ratio also fails to have an identifiable cyclical nature, in that an
The loss ratio is seen in Figure 3 to have experienced steady growth over the years. This growth cannot be expected to continue indefinitely. However, there is no statistical indication that the growth has abated. Table 1 presents the results of a multiple linear regression of LR, the loss ratio. The included years are 1939 to 1993. The independent variables are T (time in years, with T = 1 in 1939 and T = 55 in 1993) and T^2, the square of time. The T^2 term is important because if it is shown to be significantly negative it would indicate that the growth rate in the loss ratio is slowing as the level gets large. The regression result of Table 1 shows, however, that
the $T^2$ coefficient is not significantly different from zero. Therefore, loss ratio time series has not yet shown any indication that it is slowing from linear growth. When it eventually does, the time series model developed here would have to be adjusted.

The autocorrelation pattern of the loss ratio time series is shown in Figure 4. It has a strong autoregressive nature. This is recognized by the regular reduction in the magnitude of the autocorrelations as the size of the lag increases. There is not simply a single spiked autocorrelation that is beyond two standard errors from zero among a large number of nonsignificant autocorrelations. Such a nature would have given rise to a moving average model. Therefore, proposed models will need to fully embrace the apparent autoregressive nature of the data.
A comprehensive search was carried out to identify the appropriate time series model for the loss ratio. Such a model must capture the essential nature of the time series. This is done by reducing residual errors to a random white-noise pattern. The model should be parsimonious; addi-
tional terms in the model beyond those needed to yield residual white noise will not add explanatory power and could well leave a false impression of the nature of the time series pattern.

The reader is referred to the appendix for an explanation of general autoregressive-moving average ARMA(p,q) time series models. If \( q = 0 \), then there is no moving average component present and the remaining autoregressive nature is denoted as \( \text{AR}(p) \). Also, if \( p = 0 \) then the only existing component is moving average and the model is written as \( \text{MA}(q) \). The parameters \( p \) and \( q \) are the highest order of the included lags.

There is a strong indication of an autoregressive nature to the loss ratio time series, which comes from the autocorrelation pattern of Figure 4. Its steady decline over successive lags is typical of autoregressive processes. Therefore, the AR(1) model is considered first. The model results are given in Table 3. This model is very encouraging. Its adjusted \( R^2 \) value is .8852, even in the absence of higher-order terms. The autoregressive parameter \( \Phi_1 \) equals 1.0011 and has a strongly significant t-statistic. This essentially renders the model as \( (1 - B)Z_t = \alpha_t \), which is a random walk. The partial autocorrelation pattern of this random walk is shown in Figure 5. That portrayal shows that there still remain significantly nonzero residual autocorrelations of lags two and three. Consequently, it is concluded both that the loss ratio time series is not a simple random walk and that the AR(1) time series model is inadequate. The model is parsimonious, but still leaves the significantly nonzero residual autocorrelations.

Figure 6 is very helpful in the search for the appropriate time series model. It shows that the first three partial autocorrelations are significantly nonzero, while the others all fall within two standard errors of zero. This result points to consideration of an autoregressive \( \text{AR}(3) \) model. Table 4 shows abbreviated results from the search for the appropriate and

---

**Table 2. Comparison of Time Series Models**

<table>
<thead>
<tr>
<th>Model</th>
<th>Adjusted R-Squared</th>
<th>Sum of Squared Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>.8852</td>
<td>709.8</td>
</tr>
<tr>
<td>MA(1)</td>
<td>.6633</td>
<td>67531.1</td>
</tr>
<tr>
<td>AR(2)</td>
<td>.7797</td>
<td>1408.0</td>
</tr>
<tr>
<td>AR(3) complete</td>
<td>.8972</td>
<td>610.5</td>
</tr>
<tr>
<td>AR(3) without AR(2)</td>
<td>.8344</td>
<td>1933.8</td>
</tr>
<tr>
<td>AR(3) without AR(1) and AR(2)</td>
<td>.7238</td>
<td>1833.6</td>
</tr>
<tr>
<td>MA(3)</td>
<td>.7804</td>
<td>7434.7</td>
</tr>
</tbody>
</table>

---
parsimonious ARMA(p,q) model of the loss ratio time series. Table 5 shows the full results for the AR(3) model. The AR(1) model, discussed above, is an excellent starting point and has an adjusted R-squared value in excess of .88. The MA(1) model is clearly deficient, having an adjusted R-squared value of just .66. Neither the AR(2) model nor the AR(3) without the first- and second-order terms fits as well as the AR(1). However, the AR(3) with all three consecutive autoregressive terms included was superior to the AR(1). The adjusted R-squared value was larger and all three parameters had significantly nonzero t-statistics. Similarly, the AR(3) was superior to the AR(3) with the AR(2) term omitted. The AR(2) model is not expected to be found suitable because the data autocorrelations are significantly nonzero for lags 1, 2, and 3. The AR(2) model had an adjusted R-squared value that was substantially less than that of AR(3), which indicates that the extra term in AR(3) is needed to treat the autocorrelations of lag 3. Moreover, the $\Phi_3$ term does have a significantly nonzero t-statistic. A final check on the validity of the autoregressive nature was carried out by also creating an MA(3) model. It was clearly inferior, having an adjusted R-squared value of just .78.
Fig. 5. Partial Autocorrelations of the First Differences of the Loss Ratio Time Series.

Table 4. The AR(3) Time Series Model of the Loss Ratio

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHI 1</td>
<td>1.2186</td>
<td>0.14689</td>
<td>8.2964</td>
</tr>
<tr>
<td>PHI 2</td>
<td>-0.66934</td>
<td>0.24603</td>
<td>-2.7205</td>
</tr>
<tr>
<td>PHI 3</td>
<td>0.45202</td>
<td>0.15856</td>
<td>2.8507</td>
</tr>
</tbody>
</table>

The above parsimonious search procedure has led to the conclusion that the aggregate loss ratio has an AR(3) nature. From Table 5, the AR(3) model has significantly nonzero coefficients for each of the autoregressive terms. The F-statistic for the model is significantly nonzero, also indicating...
that the included terms are useful predictors. Admittedly, this model is not clearly superior to the AR(1) model in the comparison of adjusted R-squared values. The superiority of the AR(3) model is found in the significance of the t-statistics of all three autoregressive parameters and in the
existence of significant data autocorrelations for lags 1, 2 and 3. The AR(1)
model cannot satisfactorily address the data autocorrelation pattern.

There is always a concern about the stationarity of an ARMA(p,q)
process. The common remedy for demonstrated nonstationarity is to
obtain the first differences of the data values, which damps out trends.
Figures 1 and 2 show a clear upward drift for both net premium volume
and losses. The loss ratio, however, may not be so nonstationary. A check
on that was carried out. The first difference in the loss ratio was determined
as \( W_t = (1 - B)Z_t \), and is shown in Figure 7. An AR(2) model was sought for
\( W_t \) because it corresponds to an AR(3) process for the original data. The
third-order process was demonstrated above to be necessary since the data
autocorrelations are significantly nonzero through the first three lags. The
results are shown in Table 6. They show that the adjusted R-squared value
is very poor. First differencing of the loss ratio data was not a help in
rendering a parsimonious model.

The parameter estimation routine is seen from Table 5 to have yielded
the following values for the AR(3) autoregressive model: \( \Phi_1 = 1.2186, \Phi_2 = \)

\( \Phi_3 = \).

Fig. 7. First Differences in the Loss Ratio Time Series.
\[ \Phi_3 = .4520 \] Let the loss ratio at the end of year \( t \) be written as \( LR_t \). The loss ratio time series model may then be otherwise written as

\[ LR_t = 1.2186 LR_{t-1} - .6693 LR_{t-2} + .4520 LR_{t-3} + \varepsilon \]

Model validation is investigated by comparison of actual data values with forecasted annual values for a few years. Table 6 was developed by using the AR(3) model for the first 46 years to develop forecasts of the next nine years. In each of the nine years (1985–1993) the forecasted loss ratio was within the 95\% forecast interval from the model, which indicates that the model has adequately represented the nature of the loss ratio time series.

The failure of the AR(2) model to parsimoniously fit the data indicates that the time series does not have a straightforward cyclical nature. Cummins and Outreville (1987) examined the underwriting cycle and traced the work of previous researchers who concluded that the AR(2) representation is theoretically required in the face of lagged market responses to underwriting results. The AR(3) model has a significant lag of order three. This term will invalidate the nature of cycles that have a regular and consistent pattern. Therefore, the conclusion that the data have an AR(3) nature indicates that the time series does not have a stable and continuing periodic nature.

<table>
<thead>
<tr>
<th>Year</th>
<th>Lower Bound</th>
<th>Forecast</th>
<th>Upper Bound</th>
<th>Actual</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>82.81</td>
<td>89.52</td>
<td>96.24</td>
<td>88.80</td>
<td>−.72</td>
</tr>
<tr>
<td>1986</td>
<td>75.68</td>
<td>86.27</td>
<td>96.86</td>
<td>80.30</td>
<td>−5.97</td>
</tr>
<tr>
<td>1987</td>
<td>73.43</td>
<td>85.35</td>
<td>97.27</td>
<td>76.20</td>
<td>−9.15</td>
</tr>
<tr>
<td>1988</td>
<td>74.08</td>
<td>86.73</td>
<td>99.38</td>
<td>76.20</td>
<td>−10.53</td>
</tr>
<tr>
<td>1989</td>
<td>73.89</td>
<td>87.56</td>
<td>101.23</td>
<td>80.40</td>
<td>−7.16</td>
</tr>
<tr>
<td>1990</td>
<td>72.31</td>
<td>87.23</td>
<td>102.15</td>
<td>80.20</td>
<td>−7.03</td>
</tr>
<tr>
<td>1991</td>
<td>70.92</td>
<td>86.90</td>
<td>102.88</td>
<td>80.10</td>
<td>−6.80</td>
</tr>
<tr>
<td>1992</td>
<td>70.25</td>
<td>87.09</td>
<td>103.93</td>
<td>89.70</td>
<td>2.60</td>
</tr>
<tr>
<td>1993</td>
<td>69.73</td>
<td>87.40</td>
<td>105.06</td>
<td>78.90</td>
<td>−8.50</td>
</tr>
</tbody>
</table>
The actual loss ratio time series terminated at the end of year 55 (1993) with a value of 78.9. A Monte Carlo simulation was carried out using the AR(3) time series model to investigate the stability of the loss ratio over time. It was carried out by using the three initial time series actual values (1939, 1940 and 1941) as initial values. The AR(3) model was used along with randomly generated normally distributed white noise errors, which have a mean of zero and the standard error of the model.

The simulation was run five times, with 200 replications in each run. Thus, there were in all 1000 replications of the AR(3) time series. The Monte Carlo simulation results are shown in Table 7. The table contains the

<table>
<thead>
<tr>
<th>Simulation number</th>
<th>Frequency distribution Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

1Intervals are: Interval Loss ratio range
1 less than 10%
2 10–20%
3 20–30%
4 30–40%
5 40–50%
6 50–60%
7 60–70%
8 70–80%
9 80–90%
10 over 90%

The Nature of the AR(3) Time Series

The actual loss ratio time series terminated at the end of year 55 (1993) with a value of 78.9. A Monte Carlo simulation was carried out using the AR(3) time series model to investigate the stability of the loss ratio over time. It was carried out by using the three initial time series actual values (1939, 1940 and 1941) as initial values. The AR(3) model was used along with randomly generated normally distributed white noise errors, which have a mean of zero and the standard error of the model.

The simulation was run five times, with 200 replications in each run. Thus, there were in all 1000 replications of the AR(3) time series. The Monte Carlo simulation results are shown in Table 7. The table contains the
frequency distribution of the final expected loss ratio value for year 55. The 1000 observations were divided into ten intervals. The first interval included all values < 10 (% loss ratio). The second category was all final loss ratios between 10 and 20 percent. Continuing in this fashion, the final category was for all loss ratio values greater than 90 percent. The resulting frequencies were 18, 33, 71, 132, 160, 189, 183, 115, 55, and 44.

A chi-square test of homogeneity was carried out on the results of the simulation in order to see if the results were consistent over the five simulation runs. The hypothesis is that a single homogeneous probability distribution holds for the loss ratio of year 55. The aggregate frequencies for the ten intervals were used to obtain the probability distribution. The calculated chi-square value was 47.605. The critical chi-square value ($\alpha = .05$) was 50.998, with 36 degrees of freedom. Therefore, a hypothesis that the several simulation runs are yielding consistent results would not be rejected.

The actual final observed loss ratio of 78.9 would fit into the interval from 70 to 80, which had a frequency of 115. Note that this interval had a frequency that was only the fourth largest of the ten. The modal category was 50–60, with a frequency of 189, which suggests strongly that the underlying time series nature of the loss ratio is quite unstable. The sample mean of the grouped data was 53.95 and the variance was 413.597, giving a coefficient of variation of .3769, which indicates that the data do not cluster tightly around the modal category but disperse widely through all categories. It would not be surprising, therefore, for any single simulated loss ratio for year 55 to turn up in other than the modal interval. The fact that the actual loss ratio turned out to be in the interval from 70 to 80 likewise was not fully unexpected.

The implication of the result of the Monte Carlo simulation is that the loss ratio time series is somewhat unstable. Figures 1 and 2 show that the industry net premium volume and aggregate losses are quite similar, though not perfectly synchronous, in their growth over time. Figure 3 visually suggests that the loss ratio has no such regular nature. The Monte Carlo simulation affirms this and further demonstrates that there is no reliable point or interval of convergence of the loss ratio over time.

CONCLUSION

The time series nature of the aggregate property-casualty industry loss ratio has been studied here. It was demonstrated that the loss ratio has not yet shown an indication that it is slowing its nearly linear growth. The aggregate property-casualty industry could well receive this observation
with alarm because insolvencies would certainly follow continued unabated increase.

It was found that the time series nature is strongly autoregressive with lag of three (the AR(3) model). If stable cyclicity did exist it would be fully treated with an AR(2) model. Therefore, this finding suggests that the loss ratio does not have a stable periodic nature with a repeating cyclicity.

A Monte Carlo simulation of the loss ratio over time was developed and replicated 1000 times. It revealed that the loss ratio under AR(3) behavior is quite unstable. Repeated simulations that determined an industry loss ratio each year from 1939 to 1993 showed that the 1993 ratio was very inconsistent throughout the replications. Therefore, it would not be unexpected for the actual loss ratio for year 55 (1993) to vary widely from the modal interval of 50–60. That is what actually occurred, since the actual loss ratio for 1993 was 78.9(%).

The instability of the loss ratio is potentially quite troublesome. It is fully expected that premium volume and aggregate losses will continue to grow rapidly over time. The loss ratio, however, appears prone to drift somewhat erratically from its reference positions given from the three previous years. At the very least, it is clear that the loss ratio is not nearly constant over time. It does not appear that there will soon be a continuing reliable and stable relationship between premium volume and losses. Underwriters apparently will need to recognize that steady and consistent profits will arise only from investment activity. Industry underwriting activity would be foreseen to yield only frequent and erratic annual losses.

REFERENCES


Best, A.M. Company (various years) *Aggregates and Averages*, various years, Oldwick, New Jersey.


APPENDIX

Let a data value of a univariate time series at moment $t$ be denoted as $Z_t$. Define a backspace operator $B$, which has the property that $BY_t = Y_{t-1}$. Let the residual error in the model be written as $\varepsilon_t$. An autoregressive model of the time series is written as

$$(1 - \Phi_1B - \Phi_2B^2 - \ldots - \Phi_pB^p)Z_t = \varepsilon_t.$$ 

This is an autoregressive model of lag $p$ and is written $AR(p)$. The time series may require moving average terms of the residual error to remove serial correlation. Such a moving average model is written as

$$Z_t = (1 - \Theta_1B - \Theta_2B^2 - \ldots - \Theta_qB^q)\varepsilon_t.$$ 

This is a moving average model of order $q$ and is written as $MA(q)$.

The general time series model contains both autoregressive and moving average terms. It is written as

$$(1 - \Phi_1B - \Phi_2B^2 - \ldots - \Phi_pB^p)(1 - \Theta_1B - \Theta_2B^2 - \ldots - \Theta_qB^q)Z_t = (1 - \Theta_1B - \Theta_2B^2 - \ldots - \Theta_qB^q)\varepsilon_t.$$ 

Having both autoregressive and moving average terms, it is represented by $ARMA(p,q)$. A nonlinear search procedure is used to calculate the parameters of the model, usually by maximizing the accompanying log likelihood function. Ellis (1995) discusses this model.