Using Catastrophe-Linked Securities to Diversify Insurance Risk: A Financial Analysis of Cat Bonds*

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Abstract: Severe natural catastrophes in the early 1990s generated a lack of financial capacity in the catastrophe line of the global reinsurance market. The finance industry reacted to this situation by issuing innovative products designed to spread the excess risk more widely among international investors (risk securitization). The paper reviews these developments and emphasizes their significance with respect to the economic theory of risk exchanges. Special attention is devoted to the case of catastrophe-linked bonds, issued by ceding insurers to secure *ex post* conditional capital for the payment of claims. We analyze these new securities as financial portfolios combining a straight bond and catastrophe options. Using option pricing theory and simulation analysis in a stochastic interest rate environment, we show that investors attracted by the potential for diversification benefits should not overlook the optional features when including these securities in an asset portfolio. [Keywords : insurance, cat bonds, cat options, investment.]

INTRODUCTION

Past decades have shown an increasing severity and frequency of losses arising from natural catastrophes: earthquakes, hurricanes, floods, and large-scale fires. It is still controversial whether an increasing fre-

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quency of hurricanes and floods may be attributed to climate change (global warming), but it is clear that concentration of property in catastrophe-prone coastal areas has generated an increase in the amount of losses.[1]

Heavy losses from natural catastrophes have become a source of concern for the insurance industry.[2] The industry's financial capacity seems to have been outpaced by potential losses.[3] This is due to the fact that the risk of natural catastrophes is not widely diversifiable in an insurance context, where insurers supply coverage in well-defined business lines. Natural catastrophes tend to occur in selected areas of the globe: seismic regions and ocean coasts. Moreover, only a subset of these regions expresses a demand for insurance coverage. Thus, reinsurers are not able to disseminate the risk easily across the world. Moreover, cross-subsidization among different lines of business is not feasible in a competitive environment.

Traditionally, government intervention is called for when the market mechanism runs into difficulties. In the case of natural catastrophes, some subsidized programs, such as the National Flood Insurance Program in the United States, exist or have been proposed. However, securitization of natural catastrophes risk has also been proposed recently as a solution to the insurance capacity problem. In contrast to government programs, securitization represents an extension of the market mechanism. Specific conditional claims are issued and sold directly to financial investors. This occurred already with the launching of cat spreads at the Chicago Board of Trade (CBOT) and the issuing of cat bonds (or "Act-of-God" bonds) by investment banks on behalf of insurers or insurers' subsidiaries. Shirreff (1999) reports that some insurers and reinsurers have become very active in this area of business, not only by issuing securities, but also by providing bid-ask quotes of cat bonds on-line.

The purpose of this paper is to shed some light on the prospects for securitization by assessing the financial attractiveness of cat bonds. We depart from most of the literature on catastrophe insurance primarily by not considering catastrophe-linked securities from the point of view of the insurance industry's needs. We do not assess the potential for these securities to solve the insurance capacity gap—at least not directly. We emphasize the financial investors' viewpoint by concentrating on the particular risks to which cat-bondholders are exposed. Indeed, a proper assessment of these risks is required to prevent future delusions among investors and pave the ground for long-term solutions.

The next section briefly reviews the related literature. The following two sections present cat options and cat bonds in more detail. In the fifth section, a model for the pricing of cat bonds is presented and used to point out the main financial characteristics of these securities. The sixth section presents a numerical model of cat bond valuation and some simulation results. We analyze the sensitivity of cat bond prices to various changes in the financial environment. We find that the parameters of the time-series sample returns on cat bonds are non-stationary. The last section concludes.

RELATED LITERATURE

The difficulties faced by the insurance industry in dealing with catastrophes has stimulated the publication of many recent articles and special issues in risk and insurance journals. Some papers try to measure the capacity of insurance markets to deal with catastrophic losses (Cummins and Doherty, 1998). But most of the literature has focused on the search for alternatives or complements to traditional reinsurance (see Jaffee and Russell, 1997).

Mandatory public provision of coverage represents one alternative. It relies on the financial and fiscal ability of the State to spread losses both across many citizens and intertemporally. Such provision occurs in the United States with the National Flood Insurance Program. It was also imposed in a countries such as France, where all insureds pay an additional tax on their property-liability insurance contracts to finance a public fund for natural catastrophes (Magnan, 1995). However, mandatory government intervention is not free of problems. Prices are not adjusted to the risk, which results in subsidization of settlement in risk-exposed areas. Moreover, geographic dispersion of the risk is constrained by national frontiers.

Non-mandatory uses of government intervention also are possible. Lewis and Murdock (1996) proposed to complete a Chicago cat spread with similar contracts supplied by a federal authority to cover losses in the range of $25 billion to $50 billion (see Cummins et al., 1998, for financial pricing of these contracts). Although the proposal is interesting, it remains to be seen whether government provision of reinsurance capacity could be limited to the upper loss layers, once the system is established. Moreover, the amount of insured losses would not necessarily be independent of the existence of a federal program. Thus, the long-run ability of such a system not to interfere with private diversification of risk is problematic.

Economic theory teaches that losses that cannot be diversified away in a portfolio of risks or an insurance-reinsurance pool should be shared by economic agents according to their respective risk-tolerances (Borch, 1962). Doherty and Schlesinger (1998) have shown that some kind of mutualization of catastrophe risk along this line would be possible using innovations in contract design. But, as shown by Arrow (1963), financial markets provide an efficient mechanism for the sharing of non-diversifiable social risk. Moreover, given their international development over the past decades, they provide access to a huge pool of financial capacity.[4] For these reasons, several authors have noted that financial markets have a natural vocation to complete insurance markets in the dissemination of catastrophe risks; see Doherty (1997), Kielholz and Durrer (1997), and Smith et al. (1997).

The contribution of financial markets to the international diversification of catastrophes risk may occur in two ways. First, additional capital could be provided to insurers and reinsurers active in this line of business. This has occurred already to a certain extent with the recent creation of Bermuda-based companies specialized in this line of business. Second, as noted in the introduction, the securitization of catastrophe risks on behalf of the insurance industry could be organized.

There are reasons to believe that the second route is more promising. First, traditional equity financing means the provision of all-purpose capital. It finances simultaneously general business risk and the risk of fluctuations in the amount of insured losses. In the case of multiple-line insurers and reinsurers it means that suppliers of capital are taking shares in a ready-made portfolio of risks. They are constrained in the precise adjustment of their portfolio to their preferences and expectations. Second, equity financing represents *ex ante* capital. Unlike reinsurance, it does not provide *ex post* conditional capital—capital if needed. For this reason, it should be more expensive. Third, securitization offers direct access to the huge pool of capacity provided by the financial market.

The potential for risk securitization as a solution to the lack of adequate capacity in the catastrophe line of insurance business is therefore quite high. It is very likely that the risk will be more largely shared by financial investors in the future, if rewards are adequate, given the diversification effects obtained by mixing catastrophe-linked securities with other asset classes in a financial portfolio. So far, catastrophe risk securitization has occurred mainly using cat futures, cat options, and cat bonds. Cat futures and cat options have been analyzed in several papers, including D'Arcy and France (1992), Boose and Graham (1994), Canter et al. (1996), Cummins and Geman (1995), and O'Brien (1997). The more recently issued cat bonds were presented in Ganapati et al. (1997) and analyzed by Briys (1997). Our paper extends this literature by providing a simulation analysis of the the risks attached to cat bonds. We price a coupon cat bond and test the sensitivity of its duration, expected return, and variance of return to changes in the model parameters.

CHICAGO CAT OPTIONS

Catastrophe insurance options (and futures) were launched at the CBOT in December 1992. They were initially based on four indices of natural catastrophes compiled by the ISO (Insurance Services Office) from data recorded by 30 U.S. insurance companies. The four indices covered four geographic areas of the United States: west, middle west, east, and national. However, they were computed on a quarterly basis only, which made the derivative products relatively unattractive.

Since October 1995, the contracts have been based on a new set of indices produced by Property Claim Services (PCS). Nine indices are compiled daily: one national, five regional (west, middle west, east, northeast, southeast), and three local (California, Florida, Texas). These indices are highly representative, since they include about 80% of the U.S. cat market. Moreover, futures contracts were dropped, and plain vanilla options were replaced by European call spreads (purchase of a call combined with the simultaneous writing of a call at a higher strike price). Thus, insurers and reinsurers are able to buy coverage as they would with layers of aggregate excess-of-loss reinsurance.

In principle, these contracts provide a powerful tool to hedge catastrophe risks. Compared with traditional reinsurance treaties, they are much easier to negotiate, and they relieve the primary insurer from credit risk. However, they have met with only limited success so far. On the demand side, three reasons are proposed. First, basis risk[5] could explain limited interest by insurers. Second, lack of financial education by insurers and reinsurers could be the direct cause of limited trading. Third, U.S. regulation and accounting procedures are pointed out as the culprit by some authors, e.g., Smith et al. (1997).[6] In our view, the third argument has more relevance than the first two. But, fundamentally, the limited success of cat derivatives should not be looked for on the demand side. We believe that trading is limited mainly for lack of supply. A derivative market cannot operate properly without liquidity. But liquidity is not provided by hedgers. It is provided by speculators trading on their information-based expectations and by arbitragers taking advantage of pricing discrepancies arising between the cash and derivative markets. In the case of cat derivatives, arbitrage trading cannot take place because the underlying index cannot be replicated, and speculative trading is limited by lack of information about prospective cat losses among the financial investors. Thus, although cat spreads are a useful hedging tool, it is unlikely that the market will be able to expand very much in the future.

CAT BONDS

Cat bonds are designed in such a way that their return is contingent upon the occurrence of natural catastrophes or upon insurance claims from such catastrophes. If nothing happens during a specified period (risk period), they behave as a standard coupon or zero-coupon bond. However, they provide a higher yield than the yield obtained on normal bonds issued by similar institutions. If the risk materializes, the bondholder forgoes part of or the whole return. The coupons are not paid—or are paid partially. In the case of zero-coupon bonds, the bond is redeemed at below face value. In most cases, however, the capital initially invested is guaranteed.

When the trigger condition is a certain amount of losses, a development period is added to the risk period (as in the case of cat options), to allow precise calculation of losses. The trigger amount may be either the insurer's own losses or industry-wide losses as reflected, e.g., in the PCS index. In the former case, basis risk is eliminated, but investors face moral hazard since the insurer might be induced to inflate reported losses.

A much publicized example of cat bonds is the program of "earthquake risk bonds" conceived by the California Earthquake Authority. The program made provision for a $3.5 billion issue of 10-year bonds, with semi-annual coupons. The redemption of principal was guaranteed at face value, but the coupons were subject to risk. If an earthquake with total losses exceeding $7 billion occurred in California during a risk period of four years after issuance, interest payment was suspended for the rest of the bonds' life. If earthquake losses did not hit the trigger value during the risk period, coupons were paid normally until redemption of the bonds. Of the total amount of $3.5 billion, $2 billion had to be invested to provide for the capital at maturity and $1.5 billion had to flow into an insurance fund.[7]

Other examples of cat bonds are provided by the "Natural Catastrophe Insurance Notes" issued by AIG and by the United States Automobile Association (USAA) issue. In the AIG case, only 85% of return was promised if one earthquake occurred in the two-year bond life and only 40% if two earthquakes occurred. In the case of USAA, the fourth-largest homeowner insurer in the United States, the 1997 cat bond issue to an amount of $477 million was considered a great success. According to Quinn (1998), it oversubscribed 2.5:1, attracting investors worldwide struck by the strong coupon attached to the bonds—575 basis points over LIBOR. Shirreff (1999) lists 18 cat bond issues launched between December 1996 and December 1998, for a total of about $3 billion.

Typically, the funds are raised by a special-purpose company (called Residential Re in the USAA case), often located offshore for tax and regulation reasons. This company invests the funds in riskless securities (U.S. Treasury bonds) providing the basis for redemption at maturity. Simultaneously, it enters into a reinsurance contract with the insurer. The reinsurance premium is used to improve the bonds' return if the risk does not materialize. If it does, it is used in conjunction with the portion of the funds at risk to pay the reinsurance indemnities.

RISK ANALYSIS OF CAT BONDS

The marketing of cat bonds requires on diversification benefits, which financial investors may reap from investing a modest share of their portfolio in these contingent securities. Canter et al. (1996) stressed that the correlation coefficient of annual returns between the S&P 500 and the PCS index over the period 1949–1995 was slightly negative and insignificantly different from zero. This empirical result is in accordance with the widespread view that natural catastrophes are uncorrelated with systematic financial market risk.[8] Therefore, investment in cat bonds is claimed to improve the risk-return pattern of an investment portfolio; as argued by Kielholz and Durrer (1997), introducing these bonds into an internationally diversified portfolio of stocks and bonds shifts the efficient frontier to the left. However, as noted by Briys (1997), the same arguments were put forward some years ago to promote investment in junk bonds. Therefore, some caveats may be appropriate to prevent excess enthusiasm with respect to these new instruments.

(1) Risk-return calculations such as the one displayed by Kielholz and Durrer (1997) are based on in-sample analysis. They are subject to estimation risk due to non-stationarity in the parameters (expected returns, variances, and covariances). As practical applications of portfolio theory showed, out-of-sample analysis leads to different results.

(2) Cat bonds, like junk bonds, have optional features. For this reason, it is not appropriate to analyze their expected performance in a mean-variance framework. As Ambarish and Subrahmanyam (1989) demonstrated in the case of junk bonds, time series variations in the weight of the embedded option is an additional source of non-stationarity.

(3) Although cat bonds are essentially fixed-income securities, and although sensitivity of these securities to interest movements is generally a matter of concern, the issue of duration measurement has been neglected in publications dealing with cat bonds. However, as pointed out by Briys (1997), cat bonds have unconventional durations.

These aspects are illustrated using contingent claims modeling of cat bonds and simulation analysis in the rest of the paper.

A Contingent Claim Model for the Pricing of Cat Bonds

For the sake of simplicity, we assume constant interest rates. Cat bonds in our model are zero-coupon bonds, issued at time 0 with face value F and maturity time T. The bond payoff is contingent upon the value taken at time T by an index of damages from natural catastrophes occurring during an exposure period (risk period) ending at time $T' < T$. The index, with value $I(t)$ at time t, may be the PCS index—as in the case of the cat bond issue managed by Goldman Sachs on behalf of ACE Ltd.—or a specific natural-hazard index such as those published by Sigma. Further, the bond contract is assumed to be designed as follows. Final payoff is contingent upon the relationship between the value of the index at maturity, $I(T)$, and a trigger value, represented by K:

- If $I(T) \leq K$, the payoff is F.
- If $I(T) > K$, the payoff is $F - [I(T) - K]$. However, in this case, a minimum payoff of B is guaranteed.

Indeed, noting $V(T)$, the bond value at time T, there are three states of nature:

(1) If $I(T) \leq K$, implying $V(T) = F$;

(2) $K < I(T) < K + (F - B)$, implying $V(T) = F - [I(T) - K] > B$;

(3) $I(T) \geq K + (F - B)$, implying $V(T) = B$.

The end-of-period value of the bond is thus:

$$V(T) = F - \text{Max}[0, I(T) - K] + \text{Max}[0, I(T) - (K + F - B)].$$

This has the profile of a reverse call spread. Indeed, the zero-coupon cat bond is a portfolio with three components:

(1) a long position in a riskless zero-coupon bond;

(2) a short position in a cat call with strike price K;

(3) a long position in a cat call with strike price $K + F - B$.

It is straightforward to determine the present value of such a portfolio:

$$V(0) = Fe^{-rT} - C_E(I(0), K, T) + C_E(I(0), K + F - B, T) \quad (1)$$

where r is the constant interest rate, C_E represents the fair price of a European call option, and B, F, T, and K are the parameters of the cat bond contract. For given values of these parameters, equation (1) yields the fair price of the cat bond—the price that would be observed in the absence of any arbitrage opportunity in a complete financial market. As $F > B$, it turns out that $V(0) < Fe^{-rT}$.

Assume that the index follows a geometric Brownian motion in continuous time, so that: $dI(t) = \mu I(t)dt + \sigma I(t)dz(t)$. The assumption is partly justified on the ground that although catastrophes occur in discrete time and may be modeled using a Poisson process, the estimated cost of natural catastrophes fluctuates in continuous time. This makes the assumption strictly valid for the valuation of cat bonds during the period starting at time T', as no additional jump in the value of the index can occur after this date.[9] A more general model is obtained by replacing the above Wiener process by a pure Poisson process (see the next section) or a mixed diffusion process (see Cummins and Geman, 1995).[10] The main advantage of such an assumption is heuristic: it allows a closed-form solution to be derived for the cat bond price (see also Briys, 1997). In our case, the Black-Scholes (1973) pricing model is applicable.[11] The closed-form solution may be used to derive the duration of a cat bond.

The Duration of a Zero-Coupon Cat Bond

The duration measures the sensitivity of a bond to changes in interest rates. The higher the duration, the more risky the bond. For a zero-coupon bond, the duration is equal to the remaining life of the bond. This is not true for a zero-coupon cat bond. Using our pricing formula, we obtain the duration $D(t)$ at time t:

$$D(t) = -\frac{dVt}{dr}\frac{1}{Vt} = (T-t)\left\{1 + \frac{I(t)}{V(t)}[N(d_1) - N(d'_1)]\right\}.$$

As the term in brackets is greater than 1, $D(t) > T - t$. Hence, it turns out that the duration of a zero-coupon cat bond is higher than the bond life. This bond is more exposed to interest-rate risk than a standard bond. The economic reason for this result may be found in the fact that the bondholder has implicitly issued a call that gives the holder the right (but not the obligation) to buy the index at a predetermined price K. The price of a call being a positive function of the interest rate, the bondholder is exposed twice to the risk of higher interest rates: first through the long zero-coupon bond position and second, through the short call position.

This result has important implications for the attractiveness of cat bonds as a diversification vehicle in global asset allocation. Although catastrophe losses are empirically uncorrelated with financial market returns, it may be the case that cat bonds display significant positive correlation. Bond and stock returns are sensitive to interest-rate changes: higher interest rates depress stock prices as well as prices of already issued bonds.[12] As cat bond prices are more sensitive to interest-rate changes than are regular bonds, it may be expected that they will display a positive correlation with financial market returns,[13] despite their link to the catastrophe index value. Thus, their financial attractiveness is diminished.

SIMULATION RESULTS

The model presented in the preceding section is useful because it makes clear some specific features of cat bonds. However, the assumptions made to obtain a closed-form solution for the price of the bond limit its applicability. In this section, we develop a more general model for which no closed-form solution is available. For this reason, we use simulation analysis.

Distribution of Cat Bond Prices Under Stochastic Interest Rates

In this model, the cat bond payoff is driven by two random factors: the index level, generated by a Poisson process, and the interest rate, generated by a binomial process. The cat bond is a coupon bond with a coupon rate of 8%. Its lifetime is four years. Its structure is similar to the earthquake risk bonds program of the California Earthquake Authority: redemption of capital at maturity is guaranteed, but the coupons are fully at risk. If the underlying index of catastrophe losses exceeds a trigger value during the life of the bond, the remaining coupons are not paid. As in the case of the PCS index, the initial index level is set at zero. However, we drop the distinction between a risk period and a development period; the amount of losses is assumed to be known as soon as a catastrophe occurs.

The total period of four years is divided into eight half-year periods. The short-term interest rate changes at the end of each period, the change being driven by a binomial random walk process, as described in Kalotay et al. (1993). The binomial tree is constructed using the observed structure of yields to maturity on Treasury bonds with different lifetimes and a volatility measure for movements in interest rates. An example is provided in Appendix 1 for the case of four one-year periods: the short-term interest-rate volatility is assumed to be 10%, and given the strongly ascending structure of yields-to-maturity, we obtain eight distinct paths of rising interest rates.

To simulate the index of catastrophes, we first draw the number of catastrophes occurring in each subperiod from a jump distribution with parameter $\lambda = 2.2$.[14] Then the amount of losses generated by each catastrophe is drawn from a lognormal distribution with parameter $\mu = 16$ and $\sigma = 2.4$. The loss amounts are then added up, periods after periods, to obtain the index path. Given a trigger level for the index, the expected coupon payments are obtained as the product of the coupon value and the probability that the index does not exceed the trigger at the end of each year.

For each index path and each interest-rate path, the cat bond price is calculated as the present value of the guaranteed principal at maturity plus the conditional coupon payments, given a trigger level. For eight periods there are $2^8 = 256$ different interest-rate paths. Given the 10,000 simulations of the index level, we get 2,560,000 bond prices. Assuming that the index value is not correlated to financial market returns, the *ex ante* cat bond price is obtained by taking the mean of the 2,560,000 prices. This price may also be computed more directly by taking the present value of the expected cat bond payoffs along the binomial interest-rate tree. For example, using a trigger level $K = 200$ million, the unconditional probabilities of receiving the coupons are given by the following sequence: $P(I_1 < K) = 0.7769$, $P(I_2 < K) = 0.5682$, $P(I_3 < K) = 0.3851$, $P(I_4 < K) = 0.2474$. Applying these probabilities and the interest-rate path of Appendix 1 yields an *ex ante* cat bond price of 95.84.

Sensitivity Analysis

It is interesting to perform a sensitivity analysis by varying the different parameters used in the simulations, such as term structure and volatility of interest rates, expected number of catastrophes, expected severity of losses, coupon payments, and bond maturity. In general, it turns out that the bond price reacts quasi-linearly to changes in the parameters. For example, analyzing the sensitivity of the bond price to parallel shifts in the initial structure of yields-to-maturity and to changes in the coupons, it turns out, as expected, that the bond price declines as the yields-to-maturity are increased and the coupons are reduced.

The simulation analysis confirms also the theoretical result of the preceding section: the duration of a cat bond is higher than the duration of a similar risk-free bond. For both securities, the duration is a decreasing function of the interest-rate level; for a given coupon, the duration is lower when interest rates are higher (reinvestment of the coupon brings more interest payments). If the coupon is varied simultaneously, an increase (decrease) in interest rates is compensated by a decrease (increase) in the coupon.

Non-Stationarity in Cat Bonds Time Series Payoffs

As argued in the preceding section, cat bonds may be seen as portfolios composed of a riskless bond and one or several options on an index of catastrophe losses. More specifically, the coupon cat bond presented in the preceding paragraph can be decomposed into a riskless zero-coupon bond with a maturity of four years and long positions in four binomial put options on the index level paying either eight or nothing at the four respective maturities. These four options have the same strike price, given by the trigger level.

For this reason, it is inappropriate to assess the performance of cat bonds in terms of expected returns and variances. For this reason also, it is misleading to estimate these parameters using time series of realized or hypothesized returns. As time goes by, and as the probability of seeing the index exceed the trigger level changes, the weights of the options embedded in the portfolio are modified. This problem is compounded by the fact that the interest rate also changes stochastically from period to period, which also influences the prices of the embedded options and therefore their weight in the portfolio.

In Appendix 2, we present the means and standard deviations of *ex post* annual returns for the four scenarios of coupon payoffs, eight paths of annual interest rates, and a trigger level of 300 million. These values are calculated taking into account the conditional probabilities of future coupon payments as time goes by. They are compared to the corresponding values of *ex post* annual returns for the riskless coupon bond under the same interest-rate paths. As may be seen from the tables, the mean returns and standard deviations of returns are highly non-stationary from one scenario to the other. In addition, the coupon cat bond dominates the riskless coupon bond in terms of risk and return in the scenario where all coupons are paid and for four interest-rate paths in the scenario where only the first three coupons are paid (these cases are marked with an asterisk). But it is clear that *ex post* results from one sample path cannot be extrapolated to all paths.

CONCLUSION

In dealing with the natural-catastrophe risk issue, the finance industry has demonstrated its innovation capacity. Cat derivatives were first created to compensate for the shortage of financial capacity in the catastrophe line of the global reinsurance market. Cat bonds were launched more recently to avoid the liquidity problems faced by the cat derivative market. These new securities concern financial investors more directly by enlarging the universe of primary securities (stocks and bonds) accessible for global asset allocation. They make the securities market more complete.

Cat bonds represent a valuable innovation. They are a powerful tool to improve the worldwide diversification of insurance risk. They concern natural-catastrophe risk now, but the concept is applicable to other kinds of risks in the future.[15] However, the theoretical analysis and the simulation results presented in this paper show that their financial attractiveness should not be overstated. A cat bond is a combination of a straight bond and options on insured catastrophes. The option component generates an asymmetric payoff at maturity. Even if cat bonds provide a useful diversification vehicle, it is therefore inappropriate to use the standard mean-variance model to assess the risk-return performance of a portfolio including a share in cat bonds. Stochastic dominance analysis would seem more appropriate.

For the same reason, investors should not overlook that the duration of cat bonds is larger than the duration of similar straight bonds. Therefore, cat bond prices are more sensitive to movements in interest rates. It is misleading to use the absence of correlation between financial market returns and catastrophe insurance losses to infer that cat bonds are zero-beta securities. Positive correlation results from common sensitivity to interest-rate changes.

Finally, because a cat bond is a portfolio, the parameters of the time-series sample returns on this portfolio may be highly non-stationary. As time goes by, the value of the embedded options changes, and this changes the composition of the portfolio. It is therefore hazardous to assess the future performance of cat bonds on the basis of their sample distribution of returns.

Our simulation results are preliminary and will have to be confirmed by additional research. In particular, the model presented assumes no correlation between losses from catastrophes and security returns. Although this assumption is supported by past experience, it may not be valid for the future. In addition, the interest-rate process on which our results are based is unsophisticated and should be replaced by a more flexible process in future research.

Despite these caveats, we think that these results have direct relevance for prospective investors in cat bonds. They have also indirect relevance for insurers and reinsurers. The diversification potential of cat bonds is one argument, but given their specific risks, the success of these securities in providing additional capacity for coverage of catastrophe risk will depend on their yield spread. Numerical pricing of cat bonds, as used in this paper, provides a powerful tool to estimate the appropriate spread.

APPENDIX 1

Assume the observed structure of yields-to-maturity for Treasury bonds to be given by the following table.

Maturity	Yield-to-maturity
1	3.50%
2	4.20%
3	4.70%
4	5.20%

The one-year spot rate is 3.50%. At the end of the period, the binomial random walk assumption implies that it will take one of two values, r_{11} or r_{12}, with equal probabilities. These two values are linked by the interest-rate volatility parameter σ_r. We assume $r_{12} = r_{11}\exp\{2\sigma_r\}$ and $\sigma_r = 10\%$. Given this volatility and the above structure of interest rates, the bond valued using the binomial model should have a price strictly equal to the market price of a bond with the same coupon and the same maturity. This price is 100 for a two-year bond with a coupon of 4.20, for a three-year bond with a coupon of 4.70, or for a four-year bond with a coupon of 5.20. In the case of a two-year bond, the unknown future spot interest rate r_{11} is therefore the solution to the following equation:

$$100 = \frac{4.20}{1+0.035} + \frac{1}{2}\left\{\frac{104.20}{(1+0.035)(1+r_{11})} + \frac{104.20}{(1+0.035)(1+r_{11}e^{2(0.1)})}\right\}$$

This yields $r_{11} = 4.44\%$ and $r_{12} = 5.43\%$.

Repeating the process for the two subsequent periods, we get the following binomial tree:

			9.9187%
		7.01%	
	5.43%		7.5312%
3.50%		5.74%	
	4.44%		6.1660%
		4.70%	
			5.0483%

APPENDIX 2

The results in this appendix are based on the same assumptions as in the text concerning the bond contract and the stochastic process for the index. The four-year bond pays an annual conditional coupon of 8 and its redemption value is 100 with certainty. The Poisson and lognormal parameters are: $\lambda = 2.2$, $\mu = 16$, and $\sigma = 2.4$. However, the trigger level is $K = 300$ million. In addition, the T-bond yield structure recorded on October 7, 1997 is used to generate the binomial tree of interest rates.

Maturity	Yield-to-maturity
1	5.36%
2	5.65%
3	5.72%
4	6.78%

Using the above yields-to-maturity, we get the following interest-rate tree:

			14.03%
		7.11%	
	6.55%		11.49%
5.36%		5.82%	
	5.37%		9.41%
		4.77%	
			7.70%

These interest rates are first used to compute the tree structure for the default-free bond payoffs:

				108
			102.71	
		104.90		108
	107.89		104.87	
104.22		107.97		108
	111.72		106.71	
		110.60		108
			108.28	
				108

The payoffs in the table are given by the sum of the default-free bond price (computed as the expected present value of the two subsequent payoffs) and the sure coupon (8). For instance: 102.71 = [108 / (1.1403)] + 8; 104.90 = [0.5(102.71 + 104.87) / (1.0711)] + 8.

In order to calculate the cat bond payoffs under the different scenarios of coupon payments, we need to know the conditional probabilities that the index will exceed the trigger value. The reason is that these probabilities change as we proceed through the interest-rate tree, and this affects the cat bond prices (and hence the *ex post* returns). Given that $P(I_t < K | I_s > K) = 0$, for $s < t$, we get:

$$P(I_t < K | I_s < K) = \frac{P(I_t < K)}{P(I_s < K)}.$$

Using the unconditional probabilities that the index remains below 300 million, we are thus able to compute the following table of conditional probabilities:

t	1	2	3	4
$P(I_t < K \vert I_0) = 0$	0.8310	0.6637	0.4974	0.3605
$P(I_t < K \vert I_1) < K$	–	0.7987	0.5986	0.4338
$P(I_t < K \vert I_2) < K$	–	–	0.7494	0.5432
$P(I_4 < K \vert I_3) < K$	–	–	–	0.7248

The first row is used to compute the *ex ante* cat bond price, yielding 93.20.

The tree stucture for expected cat bond payoffs follows.

				102.884
			94.20	
		94.22		102.884
	96.43		96.26	
93.20		97.10		102.884
	99.97		98.01	
		99.57		102.884
			99.51	
				102.884

We use the same procedure as before, except that the expected coupon payments are substituted for a riskless coupon of 8. Hence, 102.884 = [(8)(0.3605)] + 100; 94.20 = [102.884 / 1.1403] + [(8)(0.4974)]; 94.22 = [0.5(94.20 + 96.26) / (1.0711)] + [(8)(0.6637)], and so on.

Keeping an initial price of 93.20, the conditional probabilities in the above table are then used to compute the cat bond prices and payoffs under the different scenarios of coupon payments. The second row in the table produces the cat bond prices at t = 1, contingent upon the payment of the first coupon. The third row leads to the cat bond prices at t =2, contingent upon the payment of the second coupon. The fourth row yields the cat bond prices at t = 3, contingent upon the payment of the third coupon. When the payment of the coupon is cancelled, the cat bond prices are simply given by the present values of redemption at maturity, along the various interest-rate paths. The *ex post* mean returns and standard deviations of returns are finally calculated on the basis of the prices and payoffs along each interest-rate path for each scenario of coupon payment.

With four periods, there are eight distinct interest-rate paths. They are identified in the tables below, using the following numbering of nodes in the following interest-rate tree.

			7
		4	
	2		8
1		5	
	3		9
		6	
			10

The results are as follows.

Case of default-free bond:

Interest-rate path	Mean returns	Standard deviations
1-2-4-7	6.82	4.31
1-2-4-8	6.78	3.30
1-2-5-8	6.72	3.31
1-2-5-9	6.69	2.38
1-3-5-8	6.65	3.08
1-3-5-9	6.62	2.04
1-3-6-9	6.57	2.08
1-3-6-10	6.54	0.87

Case where no coupon is paid:

Interest-rate path	Mean returns	Standard deviations
1-2-4-7	1.76	12.75
1-2-4-8	1.76	12.44
1-2-5-8	1.76	12.44
1-2-5-9	1.76	12.23
1-3-5-8	1.76	9.95
1-3-5-9	1.76	9.67
1-3-6-9	1.76	9.67
1-3-6-10	1.76	9.47

Case where only the first coupon is paid:

Interest-rate path	Mean returns	Standard deviations
1-2-4-7	3.85	10.10
1-2-4-8	3.85	9.70
1-2-5-8	3.85	7.80
1-2-5-9	3.85	7.44
1-3-5-8	3.77	10.27
1-3-5-9	3.77	10.06
1-3-6-9	3.77	8.55
1-3-6-10	3.77	8.33

Case where the first two coupons are paid:

Interest rate path	Mean returns	Standard deviations
1-2-4-7	5.95	7.63
1-2-4-8	5.93	5.88
1-2-5-8	5.87	7.96
1-2-5-9	5.87	6.69
1-3-5-8	5.79	7.82
1-3-5-9	5.79	6.53
1-3-6-9	5.74	8.15
1-3-6-10	5.74	7.20

Case where the first three coupons are paid:

Interest rate path	Mean returns	Standard deviations
1-2-4-7*	7.99	0.93
1-2-4-8*	7.95	2.51
1-2-5-8*	7.89	2.54
1-2-5-9	7.85	3.48
1-3-5-8*	7.81	2.18
1-3-5-9	7.78	3.22
1-3-6-9	7.73	3.26
1-3-6-10	7.70	4.02

Case where the four coupons are paid:

Interest rate path	Mean returns	Standard deviations
1-2-4-7*	9.92	3.62
1-2-4-8*	9.88	2.68
1-2-5-8*	9.81	2.77
1-2-5-9*	9.78	1.96
1-3-5-8*	9.74	2.52
1-3-5-9*	9.70	1.59
1-3-6-9*	9.65	1.74
1-3-6-10*	9.62	0.81

NOTES

[1] From 1970 to 1990, population in the Pacific and South Atlantic coastal states of the United States increased by 51% and 45% respectively, compared to a countrywide increase of 24% over the same period.

[2] Insurers had to pay $12.5 billion for the Northridge earthquake (1994) and $16 billion for Hurricane Andrew (1992). Hurricane Hugo (1989) cost them $5 billion, Hurricane Opal (1995) $2.1 billion, and Hurricane Fran (1996) $1.6 billion. Insured losses in excess of $1 billion has been the rule rather than the exception since the end of the 1980s; see exhibit 8 in Canter et al. (1996).

[3] The total capital of the U.S. property-casualty insurance industry is estimated at $200 billion, of which $20 billion is provided by reinsurers (Kielholz and Durrer, 1997). The coverage capacity in the catastrophe line of business (direct insurance and reinsurance) is estimated at $25 billion. This falls short of the reference losses estimated by reinsurers at $50 billion for California earthquakes and $45 billion for East Coast storms, and it is well below the maximum losses expected from these two kinds of events—$100 billion for California earthquakes and $85 billion for East Coast storms.

[4] Total capitalization of the U.S. financial market amounts to $19 trillion, with a daily standard deviation of $133 billion, which means that the typical daily fluctuation in the total value of U.S. securities is easily able to cover the maximum probable loss from a California earthquake!

[5] Basis risk occurs because the indices reflect an aggregate amount of national or regional losses. They are not necessarily strongly correlated with losses recorded by individual insurers on their own portfolios of cat risks.

[6] Some states do not allow cat derivatives to be considered a hedging tool for the definition of insurers' net liabilities. As net liabilities usually provide the basis for calculation of solvency ratios, insurers using derivatives instead of reinsurance as a hedging device are penalized by regulation.

[7] Eventually, this program was not marketed because the whole bond issue was purchased by a Bermuda-based reinsurance subsidiary of Berkshire Hathaway, which offered to reinsure the total fund of $1.5 billion for a four-year premium of $590 million (Smith et al., 1997). But it remains as a benchmark for subsequent issues.

[8] There are different opinions. As a referee pointed out, a big natural disaster might affect security returns. The point is that natural disasters are regional phenomena, whereas the financial market operates worldwide.

[9] This is not a strong assumption. As experience with the Northridge earthquake showed, the estimated cost of a major catastrophe may fluctuate widely after occurrence of the event (see Canter et al., 1996).

[10] Indeed, Cummins and Geman (1995) use a mixed diffusion process for the event period and a pure Brownian motion for the reporting period. The Poisson process represents "major" catastrophes. The Wiener process reflects small catastrophes and randomness in reporting.

[11] Under this assumption, we obtain, for $t > T'$:

$$V(t) = Fe^{-r(T-t)}[1 - N(d'_2)] - I(t)[N(d_1) - N(d'_1)] + Ke^{-r(T-t)}[N(d_2) - N(d'_2)] + Be^{-r(T-t)}N(d'_2)$$

$$d_1 = \frac{\ln(I(t)/K) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

with:

$$d'_1 = \frac{\ln(I(t)/K + F - B) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d'_2 = d'_1 - \sigma\sqrt{T-t}$$

[12] For an empirical justification, see, e.g., Jensen and Johnson (1995).

[13] Future research on cat bonds will have to test this conjecture empirically.

[14] λ represents the expected number of catastrophes per year. We chose the value of 2.2 because this is the order of magnitude of the mean number of catastrophes recorded in the United States during the period 1949–1994.

[15] However, moral hazard will set bounds on the applicability of the concept. Moral hazard is limited in the case of natural catastrophes. This would not be the same in the case of man-made catastrophes (e.g., oil spills) or in the case of social risks such as unemployment or health.

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