Measuring the Value of an Exposure: A Capital Budgeting Approach

Thomas A. Aiuppa* and Lise Graham**

Abstract: This paper presents an alternative method for evaluating property exposures, which is one part of the risk management process. The underlying premise is that the value of a property exposure depends upon the incremental cash flows lost due to a property loss and upon the firm's cost of capital; therefore, evaluating exposure should be carried out in a capital budgeting framework. Comparative analyses indicate that the exposure values produced by this method are often lower than those generated by other methods, which could in turn lead to lower insurance costs. Use of this method is both theoretically justified and consistent with a managerial focus on enhancing firm value.

INTRODUCTION

This article presents an alternative method for property exposure evaluation, which is one part of the risk management process. A property exposure has three elements: (1) the item subject to loss; (2) the perils, or forces that may cause the loss; and (3) the potential financial impact of the loss (Rodda, 1988). Measuring the financial impact is commonly referred to as "evaluating the exposure," and valuation methods include replacement cost and actual cash value. Evaluating the exposure is an important part of the firm's decisions about whether to retain losses or, if it decides to purchase insurance, how much insurance to obtain. The underlying premise of the proposed method is that the value of a property exposure depends upon the incremental cash flows due to a loss and the firm's cost

* Professor, Department of Finance, University of Wisconsin at La Crosse, La Crosse, WI.
**Associate Professor and Chairperson, Department of Finance, University of Wisconsin at La Crosse, La Crosse, WI.

Journal of Insurance Issues, 2000, **23**, 1, pp. 1–29.

of capital. The proposed method is conceptually the same as that used for capital budgeting proposals; therefore, it is called the “capital budgeting” method of exposure valuation.

Unlike other approaches, the capital budgeting method has a basis consistent with financial theory, where the emphasis is on making decisions that enhance firm value. Comparative analyses, developed and presented in the paper, indicate that the exposure values produced by this method are often lower than those generated by other methods. Consequently, if insurance is purchased based on the value of an exposure, the use of the capital budgeting method would result in lower insurance costs, which are theoretically justified. Further, retention decisions, which are also based on exposure values, would be affected. The calculations needed to implement the method include present value techniques, which can be performed on a spreadsheet. Risk managers can thus readily use the proposed method.

The paper first reviews traditional valuation standards for measuring the financial impact of a loss, then develops and discusses the proposed method. Next, the paper develops models for computing exposure values and insurance costs using various valuation standards. Representative numerical values for comparison follow. The final sections present the results and conclusions.

TRADITIONAL METHODS OF EVALUATING EXPOSURES

To evaluate property exposures, modern authors consider the lost income due to a damaged or destroyed asset. For example, Doherty (1985) considers the present value of the lost income due to the destruction of an asset and compares that value to the cost of acquiring another asset. The value of the exposure is the lower of these values.

Traditional methods to value an exposure implicitly assume that the cost of acquiring another asset is less than the present value of lost income; therefore, the value of an exposure is the cost of the asset according to some valuation method. The valuation methods include replacement cost, functional replacement cost, reproduction cost, actual cash value, and market value (Williams et al., 1995).

Replacement cost refers to the amount required to purchase a brand new, comparable asset to replace a lost, damaged, or destroyed item. A related valuation method, functional replacement cost, is the cost of acquiring an asset that, while not identical to the property being replaced, will perform the same function with equal efficiency. According to Head and

Horn (1985), this method "takes account of the danger of overstating the financial impact of a potential loss by insisting on replacement cost when the proper measure of the loss is the value of the function, not of the property itself." Reproduction cost, on the other hand, is the cost of duplicating an item, using identical materials and skills comparable to those used in the original.

Actual cash value is the replacement cost reduced by physical depreciation, obsolescence, or other considerations. Depreciation in this context is an attempt to recognize the percentage of an item's useful economic life that has been expended. As Head and Horn (1985) state, the "resulting valuation may approximate market value, because a free marketplace takes account of such factors as the replacement cost and remaining useful life of the property."

A shortcoming of the above methods is that they do not take into consideration that a firm may anticipate replacing the asset because it has a normal replacement cycle. In such cases, the firm's analysis of its potential exposure should be done using the same principles as any other capital expenditure; that is, the firm should consider the incremental cash flows associated with the event and the cost of capital needed to finance the replacement. For example, a machine may normally be replaced every twenty years. Consider a machine that is already 15 years old but could be damaged by fire today. Because the firm had planned to replace the machine in five years anyway, the firm's actual exposure is the present value of cost of acquiring the asset now minus the present value of the cost of replacing the asset at the normally scheduled time. The proposed method is thus an application of capital budgeting techniques that takes into consideration both the time until normal replacement and the firm's cost of capital.

EXPOSURE VALUES WITH THE PROPOSED METHOD

As Williams et al. (1995) note, "In risk assessment, *the objective is to estimate the economic burden of the damage on the organization's owners*." If the value of a firm is the present value of its expected cash flows, then any event or decision that affects either the timing or size of those cash flows does affect the value of the firm. When a firm expects to replace an asset at some point in the future, there is an expected cash outflow associated with that replacement. A property loss during the current year changes both the timing and the size of that outflow because the cash flow (to replace the asset) happens sooner, but the cash flow is less because the current price of the asset is less than a later, inflated price.

Thus, consistent with capital budgeting theory, which focuses on the incremental cash flows, the relevant value of a loss exposure is the difference in the present values of the cash flows under two scenarios: a loss and no loss during the current year. To calculate the present value of these cash flows, a discount rate is needed. Because the above present value problem is a capital budgeting problem, the appropriate discount rate is the firm's weighted average cost of capital.

To develop the capital budgeting method of evaluating an exposure, the following assumptions and notation are used:

Assets are normally replaced every L years, where L = life of the asset.

Currently, an asset is scheduled to be replaced in R years due to normal replacement, where $1 \leq R \leq L$. If an asset was just replaced at the end of the last year, then $R = L$.

At the end of its normal life, an asset has no residual value.

When a loss happens during a year, the asset will be replaced at the end of the current year.

C = the current value of the asset.

k = the firm's annual cost of capital.

g = the annual rate of increase in cost of an asset due to inflation.

$k > g$, which means that the cost of capital is greater than the rate of price increase.[1]

According to the capital budgeting method, the value of an exposure during a year is the present value of the cash flows if a loss happens minus the present value of cash flows if a loss does not happen. These two present-value terms are derived below.

Present Value of Cash Flows With and Without a Loss

Figure 1 shows the cash flows (represented as outflows) if a firm does not have a loss during the current year, assuming that the asset would be

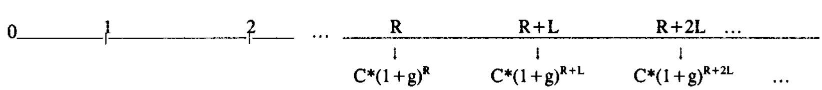

Fig. 1.

replaced in R years and then every L years thereafter.

Using the basic discounting formula, the present value (PV) of these cash flows

$$= \text{PV, if no loss} = \frac{C(1+g)^R}{(1+k)^R} + \frac{C(1+g)^{R+L}}{(1+k)^{R+L}} + \frac{C(1+g)^{R+2L}}{(1+k)^{R+2L}} + \ldots \quad (1)$$

Therefore, after factoring,

$$\text{PV, if no loss} = C\left(\frac{1+g}{1+k}\right)^R \left[1 + \frac{(1+g)^L}{(1+k)^L} + \frac{(1+g)^{2L}}{(1+k)^{2L}} + \ldots\right] \quad (2)$$

To simplify (2), consider a period to be L years. Then, the cost of capital and the rate of price increase in terms of an L-year period are as follows:

$$kk = (1+k)^L - 1 = \text{cost of capital per } L\text{-year period, and} \quad (3)$$

$$gg = (1+g)^L - 1 = \text{rate of price increase per } L\text{-year period.} \quad (4)$$

Substituting from (3) and (4) into (2) yields

$$\text{PV, if no loss} = C\left(\frac{1+g}{1+k}\right)^R \left[1 + \frac{(1+gg)}{(1+kk)} + \frac{(1+gg)^L}{(1+kk)^L} + \frac{(1+gg)^{2L}}{(1+kk)^{2L}} + \ldots\right] (5)$$

Note that the term in the brackets of (5) is 1 plus a constant growth model, where the growth rate is gg and the cost of capital is kk. Therefore, the sum of the terms in the brackets equals $[1 + (1 + gg)/(kk - gg)]$, which reduces to $(1 + kk)/(kk - gg)$. Consequently,

$$\text{PV, if no loss} = C\left(\frac{1+g}{1+k}\right)^R \left(\frac{1+kk}{kk-gg}\right) \quad (6)$$

This result represents the present value of the cash flows, if no loss occurs during the current year, and is part of the equation for the value of an exposure under the capital budgeting method. The other part of the formulation is the present value of cash flows if the firm has a loss during the current year. In that case, the firm would replace the asset at the end of the year, and then again every L years. Under this scenario, the cash flows and their present value are as represented in Figure 1 and in (6), respectively, but R is replaced with 1. Therefore,

$$\text{PV, if a loss} = C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right) \tag{7}$$

The value of an exposure under the proposed method is defined as the difference of present values of cash flows with and without a loss. That difference is found by subtracting equation (6) from equation (7) and is denoted by EXP(CB). Therefore,

$$\text{EXP(CB)} = C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right) - C\left(\frac{1+g}{1+k}\right)^R\left(\frac{1+kk}{kk-gg}\right) \tag{8}$$

$$= C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\left[1-\left(\frac{1+g}{1+k}\right)^{R-1}\right] \tag{9}$$

The expression in (9) does not hold for $k = 0$ (because it was based on the result for a constant growth model, which necessitates that $k > 0$). Intuitively, if the cost of capital equals 0, the value of the exposure equals 0 because a new asset can be purchased from funds that cost the firm nothing to acquire. Analytically, this result can be obtained by first listing out the individual terms of the difference of the present value of the cash flows with a loss and the present value of cash flows without a loss. Then, after similar terms of opposite signs cancel out, a finite number of terms remain to be summed. When these terms are evaluated with $k = 0$, their sum equals 0.

EXPOSURE VALUES WITH TRADITIONAL APPROACHES

In this section, the values of an exposure are formulated for two traditional methods: replacement cost and actual cash value. The algebraic expressions for exposure values are subsequently used for insurance cost comparisons, but could also be used for retention decisions.

Value of an Exposure Under Replacement Cost

If the firm evaluates the exposure on the basis of a replacement cost basis, the value of the exposure would be the replacement cost as of the end of the year. That value is represented by EXP(REP), and

$$\text{EXP(REP)} = C\,(1+g). \tag{10}$$

Value of an Exposure Under Actual Cash Value

Actual cash value (ACV) is defined as replacement cost less depreciation. Because depreciation accounts for such things as physical deterioration and obsolescence, several depreciation schedules are possible. In this paper, the depreciation is assumed to be a constant percentage each year.[2] Consequently, the ACV would be formulated as the number of years remaining minus one, divided by the life of the asset times the replacement cost of the asset as of the end of the year. The number of years remaining minus one is used because the firm expects to replace the asset R years from now. For example, if the asset life is 10 years, if the asset has four years until normal replacement, and if the cost of the asset is $C\,(1 + g)$ as of the end of the year, then the exposure value according to ACV is $(10 - 4 - 1)\ /\ 10$ times $C\,(1 + g) = 0.3\,C(1 + g)$. The value of the exposure under ACV is represented by EXP(ACV), and

$$\mathrm{EXP(ACV)} = \frac{R-1}{L}C(1+g) \tag{11}$$

Note that during the last year of the asset (when $R = 1$), the value of the exposure is 0, because the firm plans to replace the asset, whether or not a loss occurs. Other depreciation schedules can be similarly modeled.

Results of the calculations of the exposure values under the various methods are presented after the resulting insurance costs are discussed.

INSURANCE COSTS

When a firm decides to purchase insurance on an asset, it chooses a limit based on the value of the exposure. In this section, the insurance costs under different valuation approaches are developed, and the next section contains the numerical values for comparisons. An underlying assumption of these analyses is that replacing a destroyed asset is economically profitable for the firm. That is, the present value of the expected cash flows generated by the replaced asset do exceed the cost of the asset.

For each valuation method, two types of insurance costs are defined. One value is the insurance cost for the current year, and that value is called the "annual cost." The other value is the PV of insurance costs over an infinite time horizon, and that cost is called the "aggregate cost." The rationale for the aggregate cost is to evaluate the long-term effects of each method of exposure evaluation. Further, the intermediate steps to develop the aggregate costs can be used to calculate the costs over a finite time period.

Annual Insurance Costs

To formulate the annual cost analysis, a standard convention is reviewed. The insurance premium for any year equals α times the value of the exposure, where α is the rate per dollar of coverage. With this convention, the annual insurance cost for each method of exposure evaluation is simply α times the exposure values already derived. Consequently, from (9), (10), and (11), respectively:

Annual insurance cost from the capital budgeting method

$$=\mathrm{ANN(CB)} = \alpha\mathrm{EXP(CB)} = \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\left[1-\left(\frac{1+g}{1+k}\right)^{R-1}\right] \tag{12}$$

Annual insurance cost from the replacement cost method

$$= \mathrm{ANN(REP)} = \alpha\mathrm{EXP(REP)} = \alpha C(1+g). \tag{13}$$

Annual insurance cost from the ACV method

$$= \alpha\mathrm{EXP(ACV)} = \alpha\frac{R-1}{L}C(1+g) \tag{14}$$

Aggregate Insurance Costs

To calculate the aggregate insurance costs, the discounted annual costs need to be summed over an infinite time horizon. To formulate the aggregate cost analyses, two issues need to be discussed. The first issue is a timing convention. The exposure during any year is evaluated during the previous year, and the insurance for the exposure is purchased at the end of the previous year. For example, during year 2000, the risk manager would evaluate the exposure for 2001. To be on the conservative side and to comply with coinsurance provisions for property insurance, the exposure value would be based on the value of the asset as of the end of 2001. Then the insurance with a limit equal to the year-end 2001 exposure value would be purchased at the end of 2000.

The second issue is that, as of the current year, the exposure during a subsequent year depends on whether a loss happens during the current year. As of the beginning of the current year, the asset has R years left until replacement, and the exposure and insurance cost are based on that value of R. As of the beginning of the of the next year, the asset will have $R-1$ years left, if the firm does not have a loss during the current year. However,

Table 1. Notation for Exposure Values and Insurance Costs

Method	Value of Exposure	Annual Insurance Costs	Aggregate Insurance Costs (No Losses)	Aggregate Insurance Costs (Losses)
Capital Budgeting	EXP(CB)	ANN(CB)	AGGN(CB)	AGGL(CB)
Replacement Cost	EXP(REP)	ANN(REP)	AGG(REP)[a]	AGG(REP)[a]
ACV	EXP(ACV)	ANN(ACV)	AGGN(ACV)	AGGL(ACV)

[a]With replacement cost, the aggregate insurance costs are equal under the scenarios of no loss and loss.

if the firm has a loss during the current year, and the asset is replaced (as of the end of the year), the asset will have L years left. Consequently, the exposure value during year 2 is based on either $R - 1$ or L years. Similarly, the exposure value for year 3 is based on the loss possibilities of years 1 and 2, and so on. However, if the exposure is measured by replacement cost, the insurance cost is always based on the cost of replacing the asset. That is, the value of the exposure is the same whether or not a loss happened during the previous year or years.

In the following sections, the aggregate insurance costs will be determined for the replacement cost method, where the loss assumption is not relevant, and also for for the capital budgeting and ACV methods under both scenarios (no loss; loss). The no-loss scenario would appear to obviate any sort of insurance cost analysis because if the firm will have no losses, insurance is not necessary. However, that scenario is used to generate a base case for comparisons. Table 1 contains the notation for exposure values and insurance costs.

Aggregate Cost Under the Capital Budgeting Method—Without Losses

Under the assumption of no losses, where the exposure is measured by the capital budgeting method, the aggregate insurance costs, denoted by AGGN(CB), are as follows:

$$\mathrm{AGGN(CB)} = \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\left[F(R) + \left(\frac{1+g}{1+k}\right)^{R}\left(\frac{1+kk}{kk-gg}\right)F(L)\right] \quad (15)$$

where

$$F(R) = \left[1 - \left(\frac{1+g}{1+k}\right)^{R}\right]\left(\frac{1+g}{k-g}\right) - R\left(\frac{1+g}{1+k}\right)^{R-1} \tag{16}$$

and $F(L)$ is as $F(R)$ in (16), but R is replaced with L. The derivation is contained in Appendix I.

Aggregate Cost Under Replacement Cost

If replacement cost is used as a measure of the exposure, and if insurance is purchased equal to the value of the exposure, the annual insurance cost, ANN(REP), was shown to equal αEXP(REP) = $\alpha C(1 + g)$. Consequently, the aggregate insurance cost, AGG(REP), is the sum of discounted costs over an infinite time horizon. Further, the cost of the asset increases each year by a factor of $(1 + g)$ to account for price increases. Therefore,

$$\text{AGN(REP)} = \alpha C(1+g) + \alpha C\left(\frac{(1+g)^2}{(1+k)^2}\right) + \alpha C\left(\frac{(1+g)^3}{(1+k)^3}\right) + \ldots$$

$$= \alpha C(1+g)\left[1 + \frac{(1+g)}{(1+k)} + \frac{(1+g)^2}{(1+k)^2} + \ldots\right] \tag{17}$$

Because the terms within the brackets of (17) equal 1 plus a standard growth model, they can be replaced with $1 + (1 + g)/(k - g) = (1 + k)/(k - g)$. Consequently,

$$\text{AGG(REP)} = \alpha C(1+g)\left(1 + \frac{1+g}{k-g}\right) = \alpha C(1+g)\frac{1+g}{k-g} \tag{18}$$

Note that this value holds under both scenarios—no loss and loss.

Aggregate Cost Under the Actual Cash Value—Without Losses

Under the assumption of no losses, where the exposure is measured by the ACV method with a constant depreciation percentage, the aggregate insurance costs, denoted by AGGN(ACV), can be shown to be

$$\text{AGGN(CB)} = \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2}\left[(R-1)H(R) + (L-1)\frac{(1+g)^R}{(1+k)^R}\left(\frac{1+kk}{kk-gg}\right)H(L)\right] \tag{19}$$

where

$$\text{H(R)} = \left[\frac{(1+g)^R}{(1+k)^R} + R\left(\frac{k-g}{1+k}\right) - 1\right] \tag{20}$$

and $H(L)$ is like $H(R)$ in (20) with R replaced by L. The derivation is found in Appendix II.

Aggregate Insurance Costs Under the Assumption of Losses

In the previous sections, the aggregate insurance costs were presented under the assumption that a firm would not incur losses. In this section, the aggregate costs are considered under the more realistic assumption that firms can have a loss during any year. The change in assumption affects the values for the annual insurance cost under the capital budgeting method and the ACV method, and, therefore, affects the aggregate insurance cost under these methods. As previously explained, for the replacement cost method, the annual costs, and therefore the aggregate cost of insurance, are the same under both assumptions. Before the specific results are presented, some general discussion is included for both the capital budgeting method and the ACV method.

Let E_0= cost of insurance for the first year, and that cost is paid at the present time.

The value of E_0 is known with certainty; further, it does not need to be discounted because it is paid at the beginning of the year, and it is based on an asset cost of $C(1+g)$ because the asset is insured for its value as of the end of the year. Then, let E_t (for $t \geq 1$) = expected value of the present value of the cost of insurance for year $t+1$, where the cost is paid at time = t. Each E_t ($t \geq 1$) is an expected value because the insurance cost for each year after year 1 depends upon whether losses will have happened in previous years. To incorporate the expected value, an assumption is made that the probability of total loss to the asset in any year is π.

Consider the expected value of the present value of insurance cost for year 2, paid at the end of year 1. That value is E_1, and

Table 2. Scenarios and Parameters for the Expected Cost of Insurance for Year 3

Scenario	Probability	Remaining life of asset as of beginning of year 3
1. No loss in year 1, and no loss in year 2	$(1 - \pi)^2$	$(R - 2)$[a]
2. No loss in year 1, and a loss in year 2	$(1 - \pi)\pi$	L
3. A loss in year 1, and no loss in year 2	$\pi(1 - \pi)$	$L - 1$
4. A loss in year 1, and a loss in year 2	π^2	L

[a]If $R - 2 = 0$, then the asset would have been replaced with a new asset, and the remaining life is L years.

$$E_1 = \pi^*(\text{insurance cost for year 2 for an asset with } L \text{ years remaining, paid at t} = 1)/(1 + k) + (1 - \pi)^* (\text{insurance cost for year 2 for an asset with } R - 1 \text{ years remaining, paid at } t = 1)/(1 + k) \tag{21}$$

Note that the first term of (21) above refers to the case where the firm had a loss during the first year, and the asset was replaced with a new asset whose life is L years. The second term refers to the no-loss situation, so the remaining life is $R - 1$, which reflects one less year of life. (However, if $R - 1 = 0$, the asset would have been replaced with a new asset whose life is L years.) Further, the insured value of the asset is $C(1 + g)^2$.

Now consider E_2 = expected value of the present value of the insurance costs for year 3, where the cost is paid at the end of year 2. That expected value has four terms, reflecting the scenarios that could happen during the previous years. The scenarios and their relative parameters are given in Table 2.

The probability associated with each scenario is a product of probabilities of the events (loss or no loss) in each scenario because the losses during the preceding years are assumed to be independent. The remaining life of the asset as of the beginning of year 3 follows directly from the description of the scenario. For example, in scenario 2, there was no loss in year 1, but a loss in year 2. Therefore, the asset will be replaced at the beginning of year 3 with a new asset whose life is L years. Under scenario 3, a loss could happen in year 1, and the asset would be replaced with an asset whose life is L years. Then, no loss could happen in year 2, and the asset would have $L-1$ years remaining as of the beginning of year 3. An additional comment is needed for scenario 1: If, at the beginning of a year, $R-2=0$, that means that the asset will be replaced, and the remaining life becomes L years.

The expected value, E_2, is calculated analogously to that of (21), but the asset cost for insurance purposes is $C(1+g)^3$ and each term is discounted by $(1+k)^2$. In general, the expected value of the present value of insurance cost for year t+1, which is paid at the beginning of year t, will have 2^t terms. To compute the expected present value when t is large would take a considerable number of calculations. For example, if $t = 25$, the calculations would involve 2^{25} terms, and that number is greater than 33 million. However, an iterative approach is possible. The iterative approach is presented below, without proof; however, the proof is available from the authors.

The iterative method to calculate each E_t for $t \geq 2$ is

$$E_t = \frac{1+g}{1+k}\left\{E_{t-1} - (1-\pi)^{t-1}\left(\frac{1+g}{1+k}\right)^{t-1}\left[ANN_X - (1-\pi)ANN_Y - \pi ANN_Z\right]\right\} \tag{22}$$

where ANN refers to the annual cost of insurance for the *current year* under any of the described methods of exposure evaluation. That is, if the analysis is intended for the capital budgeting method, ANN is replaced with $ANN(CB)$ as defined in (12), and if the analysis is in terms of the ACV method, then ANN is replaced with $ANN(ACV)$ from (14). The subscripts of ANN indicate the number of years of remaining life to be used in the calculation. The values of the subscripts are

$$X = \begin{cases} R-t+1 & \text{if } 2 \leq t \leq R \\ L-[(t-1-R)] \text{ MODL} & \text{if } t \geq R+1 \end{cases}$$

$$Y = \begin{cases} R-t & \text{if } 2 \leq t \leq R-1 \\ L-[(t-R)] \text{ MODL} & \text{if } t \geq R \end{cases}$$

$$Z = L-[(t-1) \text{ MOD L}]$$

where MOD is the modular function. The MOD function, a MOD b, returns the remainder when a is divided by b.

Recall that each E_t equals the expected present value of cost at time t. Consequently, the aggregate insurance cost equals the sum of the values of E_t from $0 \leq t \leq$ infinity. This sum will converge because each E_t contains terms of the form $1 / (1+k)^t$, and for large values of t, these terms will tend to 0. In this analysis, the terms E_t are summed over blocks of L terms including the first R values. That is,

$$S_0 = \sum_{t=0}^{R-L-1} E_t,\ S_1 = \sum_{t=R+L}^{R-2L-1} E_t,\ S_2 = \sum_{t=R+2L}^{R+3L-1} E_t,\ \ldots,\ S_j = \sum_{t=R+jL}^{R+(j+1)L-1} \tag{23}$$

and convergence is determined when the absolute value of the percentage change in two consecutive S values is less than or equal to .0001. That is, until

$$\left| \frac{S_{j+1} - S_j}{S_j} \right| \leq .0001$$

Note that the above process generates the expected aggregate insurance cost over an infinite time period. If the expected aggregate cost over a finite time period is desired, the terms E_t can be summed just over that period.

RESULTS

Representative results are presented in Tables 3 and 4. In both tables, the cost of capital and the rate of price increase are .10 and .05, respectively. Further, the current cost of the asset, C, is \$100, and the insurance rate, α, is \$0.01 per \$100 of property value. For other values of the current cost and the insurance rate, the values within the table can be modified easily by multiplying by the appropriate factor. However, the choice of the values for the two latter parameters serves another purpose; the annual insurance costs computed with these values also equals the value of the exposure per each \$1 of current asset value.

Effects of the Probability of Loss on the Aggregate Insurance Costs

Table 3 is intended to show the effect of the probability of loss on the values of the aggregate insurance costs under both the capital budgeting and the ACV methods. Values for the replacement cost method are not presented in the table because they do not depend on the probability of loss. The values in the columns labeled "$\pi = 0$" refer to the case when the probability of loss in each year equals 0, and the resulting aggregate insurance costs in the column were calculated directly as AGGN(CAB) and AGGN(ACV) from (15) and (18), respectively, where the probability of loss was assumed to be 0. The values in the columns when $\pi > 0$ are expected values for the aggregate insurance costs when the probability of loss is greater than zero and were calculated iteratively as AGGL(CAB) and AGGL(ACV) from (23). Each value in parentheses represents the percentage error between the insurance cost calculated with the probability of loss equal to 0 and calculated with a probability of loss greater than 0. For example, for the capital budgeting method, when $L = 10$ and $R = 1$, the aggregate insurance cost is \$10.566 if the probability of loss equals 0. However, if the cost is calculated with a probability of loss equal to 0.01, the aggregate insurance cost is \$10.732. Consequently, the value under the no-loss scenario is 1.57 percent too low. The results of Table 3 indicate that if one uses the formulas based on no losses, when in fact the probability of loss is greater than 0, the resulting values can be substantially different, and the error increases as both the life of the asset and the probability of loss increase.

Table 4 contains the annual and aggregate insurance costs based on the capital budgeting, ACV, and replacement cost methods. For the aggregate costs based on the first two methods, the probability of loss is assumed to be 0.01, and the values were calculated by the iterative formula (23).

Annual Insurance Costs

When the results for the annual costs are reviewed, some expected findings are seen. When $R = 1$, the annual insurance costs equal 0 under both the capital budgeting and the ACV methods; because the firm plans to replace the asset because of the normal replacement schedule, it does not insure the asset that year. For all values of R and L, the annual insurance cost under the replacement cost method is constant, and that cost is higher than the costs under the other methods because the firm would be insuring the asset for the cost of a new asset. Further, when the asset is not necessarily going to be replaced at the end of the current year (when $R \geq 2$), the insurance cost under replacement cost can be several times more than the costs under the other methods. Additionally, the shorter the time to scheduled replacement (that is, the lower the value of L), the greater the difference between the replacement cost value and the other values.

An important result from the calculations is that the annual insurance cost is generally larger under the capital budgeting method than under the

Table 3. Aggregate Insurance Costs for Values of P (Loss)

Remaining Life	Method for Aggregate Insurance Costs	Asset Life $L = 10$ P(Loss) $\pi = 0.00$	$L = 10$ $\pi = 0.01$	$L = 10$ $\pi = 0.05$	$L = 20$ P(Loss) $\pi = 0.00$	$L = 20$ $\pi = 0.01$	$L = 20$ $\pi = 0.05$	$L = 50$ P(Loss) $\pi = 0.00$	$L = 50$ $\pi = 0.01$	$L = 50$ $\pi = 0.05$
$R = 1$	Capital Budgeting	10.566	10.732 (–1.57%)	11.399 (–7.88%)	12.570	12.881 (–2.47%)	14.057 (–11.83%)	16.489	16.966 (–2.89%)	18.275 (–10.83%)
	ACV	10.766	10.946 (–1.67%)	11.674 (–8.43%)	12.155	12.504 (–2.87%)	13.849 (–13.94%)	14.735	15.417 (–4.63%)	17.484 (–18.66%)
$R = 2$	Capital Budgeting	10.202	10.366 (–1.60%)	11.023 (–8.05%)	12.071	12.373 (–2.50%)	13.522 (–12.02%)	15.788	16.251 (–2.93%)	17.534 (–11.06%)
	ACV	10.381	10.558 (–1.71%)	11.275 (–8.61%)	11.655	11.994 (–2.91%)	13.304 (–14.15%)	14.086	14.744 (–4.67%)	16.750 (–18.91%)
$R = 10$	Capital Budgeting	11.069	11.243 (–1.57%)	11.941 (7.88%)	10.799	11.142 (–3.18%)	12.477 (–15.54%)	12.546	13.083 (–4.28%)	14.816 (–18.09%)
	ACV	11.278	11.467 (–1.68%)	12.230 (–8.44%)	10.095	10.470 (–3.71%)	11.954 (–18.42%)	10.534	11.203 (–6.35%)	13.486 (–28.02%)

Cost of capital = 10%
Rate of price increase = 5%
Current asset cost = $100
Insurance rate = $0.01 per $100 of value

Table 4. Annual and Aggregate Insurance Costs

		Asset Life							
		L = 10		*L* = 20		*L* = 50		*L* = 100	
		Insurance Cost		Insurance Cost		Insurance Cost		Insurance Costs	
	Method	Annual	Aggregate	Annual	Aggregate	Annual	Aggregate	Annual	Aggregate
	Cap. Budgeting	0.00	10.372	0.000	12.881	0.000	16.966	0.000	19.531
R = 1	Replacement	1.050	23.100	1.050	23.100	1.050	23.100	1.050	23.100
	ACV	0.000	10.946	0.000	12.504	0.000	15.417	0.000	18.118
	Cap. Budgeting	0.117	10.365	0.072	12.373	0.048	16.251	0.044	18.698
R = 2	Replacement	1.050	23.100	1.050	23.100	1.050	23.100	1.050	23.100
	ACV	0.105	10.558	0.052	11.994	0.021	14.744	0.000	17.313
	Cap. Budgeting	0.878	11.243	0.539	11.142	0.362	13.083	0.330	14.663
R = 10	Replacement	1.050	23.100	1.050	23.100	1.050	23.100	1.050	23.100
	ACV	0.945	11.467	0.472	10.470	0.189	11.203	0.094	12.613
	Cap. Budgeting			0.894	13.164	0.600	12.808	0.547	13.613
R = 19	Replacement			1.050	23.100	1.050	23.100	1.050	23.100
	ACV			0.945	12.674	0.378	10.042	0.189	9.990

Cost of capital = 10%
Rate of price increase = 5%
Current asset cost = $100
Insurance rate = $0.01 per $100 of value
The annual insurance cost also equals the exposure value.

ACV method (assuming a constant depreciation percentage), except when the value of R approaches the value of L. For example, as noted in Table 4, when $R = 10$ and $L = 10$, and when $R = 19$ and $L = 20$, the capital budgeting value is less than the ACV value. Although the values in Table 4 were calculated for certain parameter values (P(Loss) = .01, rate of price increase = 5 percent, and cost of capital = 10 percent), the relationship just noted holds over a wide range of values of the parameters. Consequently, in general, the capital budgeting values are in between the values for the ACV and replacement cost methods, but the capital budgeting values are closer to the ACV values.

As indicated previously, the annual insurance costs calculated with the current cost of the asset equal to \$100 and the insurance rate equal to \$0.01 per \$100 of asset value also equals the exposure value per \$1.00 of current asset value. Consequently, the values in the column for annual insurance costs represent exposure values, and the same findings apply. That is, the exposures measured under the capital budgeting approach are generally in between the measures of the ACV and replacement cost methods, but are closer to the ACV values. The exposure values can be used for retention decisions. Firms that are currently basing their retention decisions on replacement cost values may be more likely to retain losses after considering the exposure values in a capital budgeting framework.

Aggregate Insurance Costs

The aggregate cost of insurance is computed as the present value of the insurance costs over an infinite time period and is used to evaluate the effect of using the various valuation methods over a long time horizon. The aggregate costs as reported in Table 4 parallel the results for the annual costs. The replacement cost method produces the highest insurance costs among all the methods presented. Further, the capital budgeting values are generally between the ACV values and the replacement cost values, but they are closer to the former.

The magnitude of the relative costs produced by the replacement cost method and the capital budgeting method is important to note. As seen in Table 4, the resulting aggregate insurance cost under replacement cost can be two times as high as the cost under the capital budgeting method. Consequently, firms that are currently basing insurance purchases on exposure values according to replacement cost are incurring higher insurance costs than are theoretically justified.

SUMMARY AND CONCLUSIONS

This paper presents a method for evaluating property exposures based on the premise that the value of an exposure is measured by the impact of a loss on the firm's cash flows. The proposed method is intended for risk managers deciding whether to retain losses or considering how much insurance to purchase. The technique is developed for assets with no residual value that are routinely replaced (for example, certain types of equipment, improvements, and betterments that have no residual value). However, the methodology can be modified to account for assets that do have an expected residual value.

The results indicate that a commonly used valuation technique, replacement cost, can result in exposure estimates that are substantially greater than the financial impact of a loss. Consequently, firms buying insurance equal to the replacement cost may be overinsured, thereby incurring higher expenses than are theoretically justified and reducing the value of the firm. (The values under the proposed method are indeed closer to the values produced by an ACV evaluation with a constant depreciation percentage.)

The calculations under the proposed method for the value of an exposure and for the resulting insurance costs for an upcoming year (the annual insurance costs) are easily adaptable to a spreadsheet or programmable financial calculator. Although the aggregate insurance costs (the present value of insurance costs over an infinite time horizon) can be more tedious, they are not necessary to make an insurance purchasing decision for the coming year. The information for the aggregate insurance cost is useful to demonstrate the long-run cost savings if a firm changes the method by which it evaluates exposures.

The technique also should be understood by members of the board of directors, insurance companies, and creditors in addition to risk managers. Managers may need to work with insurers to develop a procedure for "dividing an asset" for insurance purposes. For example, the firm may need to separately insure the shingles of a roof (which may have a 20-year life) from the rest of the building (which may have a 100-year life). Firms may also have to negotiate insurance policies to adjust or to remove coinsurance clauses because, under the proposed technique, insurance would not always be purchased equal to some stated coinsurance percentage. Additionally, managers will need to consider potential creditor requirements that insured values be at least equal to the outstanding balance of the financing for the asset.

Managers may initially be reluctant to purchase insurance for less than replacement cost value because, if a loss happens, they may prefer to say that the loss was fully covered rather than having to justify the purchase of less insurance. A board-approved risk management policy that endorses the use of the capital budgeting method to evaluate exposures and subsequently determine insurance values would provide support for management in these instances. Furthermore, board members should favor such a policy, since it is consistent with a managerial focus on enhancing firm value.

APPENDIX I

Derivation of Aggregate Insurance Costs under the Capital Budgeting Method (No Losses)

Under the scenario of no losses, the aggregate insurance cost under the capital budgeting method, AGGN(CB), is the sum of discounted values of the annual insurance costs over an infinite time horizon. Consequently, before the derivation is presented, it is necessary to discuss the algebraic representation of insurance costs to be paid at future times. To represent the annual costs for future time periods in terms of the values of the parameters for the current time period, some adjustments need to be made. First, the value of the asset needs to be increased by a factor of $(1+g)$ each year to account for price increases. (For example, if the asset cost is $C(1+g)$ at the end of the first year, then its cost is $C(1+g)^2$ at the end of the next year.) Second, the number of years of remaining life needs to be decreased by 1 year for each year in the future. A final adjustment is that when the remaining life of the asset becomes zero, the asset will be replaced with a new asset whose life is L years. Consider the annual insurance costs under the ACV method for an asset whose current cost is C, whose value increases by g percent each year, and whose remaining life is $R = 2$ years. Further, when the asset is replaced, the new asset will have a life of $L = 10$ years. The annual insurance costs over a four-year time period, in terms of the current parameters, are presented in Table A; the cost formula is based on (14) and on the adjustments just discussed.

In general, if ANN_R equals the annual cost of insurance for the current year (as measured by any of the methods) when an asset has R years remaining, then

$$\text{the annual cost of insurance for year t equals } (1+g)^{t-1}\,\mathrm{ANN}_X, \qquad \text{(A1)}$$

where

Table A. Example of Annual Insurance Costs over a Four-Year Time Period, ACV Method

For Year	Remaining Life of Asset	Annual Insurance Costs (ACV Method) (In terms of current parameters)
1	$R = 2$	$\alpha\frac{R-1}{L}C(1+g) = \alpha\frac{2-1}{10}C(1+g)$
2	$2 - 1 = 1$	$\alpha\frac{(R-1)-1}{L}C(1+g)^2 = \alpha\frac{2-1}{10}C(1+g)^2$
3	$L = 10$	$\alpha\frac{L-1}{L}C(1+g)^3 = \alpha\frac{10-1}{10}C(1+g)^3$
4	$10 - 1 = 9$	$\alpha\frac{(L-1)-1}{L}C(1+g)^4 = \alpha\frac{(10-1)-1}{10}C(1+g)^4$

Current asset cost = C,
Rate of price increase = g,
Life of new asset = $L = 10$
Remaining life = $R = 2$

$$X = \begin{cases} R - t + 1 & \text{if } t \le R \\ L - (t - 1 - R)\ MODL & \text{if } t \ge R + 1 \end{cases}$$

and where MOD is the modular function. The value of a MOD b equals the remainder when a is divided by b.

To begin the derivation of the aggregate insurance cost, first consider the sum of the discounted insurance costs over the first R years, and let t index the number of years since the present time ($0 \le t \le R - 1$). That partial summation is

(Annual insurance cost with R years remaining, paid at $t = 0$) / $(1 + k)^0$

+(Annual insurance cost with R–1 years remaining, paid at $t = 1$) / $(1 + k)^1$

+...

+ (Annual insurance cost with 1 year remaining paid, at t = R–1) / $(1 + k)^{R-1}$ (A2)

Note that the first discount factor has a zero exponent because that insurance cost is paid at the beginning of the year.

Consider the annual insurance cost paid at t = 0, the beginning of year 1. From (12), that cost is

$$\mathrm{ANN(CB)} = \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\left[1-\left(\frac{1+g}{1+k}\right)^{R-1}\right] \tag{A3}$$

As previously discussed, to represent the annual insurance cost for the next year *in terms of the current values of R and C*, two adjustments need to be made. First, R needs to be replaced with R–1 because for the next year, the asset has one less year until normal replacement. Second, C needs to be replaced with $C(1 + g)$ because as of the next year, the price of the asset increases by g percent. Likewise, similar adjustments are made for subsequent years.

Consequently, the present value (PV) of the insurance costs over the first R years is

$$\begin{aligned} &\alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\left[1-\left(\frac{1+g}{1+k}\right)^{R-1}\right] + \\ &\alpha C(1+g)\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{1+gg}\right)\left[1-\left(\frac{1+g}{1+k}\right)^{R-2}\right]/(1+k) + \\ &\ldots + \alpha C(1+g)^{R-1}\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\left[1-\left(\frac{1+g}{1+k}\right)^{1}\right]/(1+k)^{R-1} \end{aligned} \tag{A4}$$

After factoring, this sum equals

$$\begin{aligned} \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\Bigg\{&\left[1-\left(\frac{1+g}{1+k}\right)^{R-1}\right] + \left[\left(\frac{1+g}{1+k}\right)-\left(\frac{1+g}{1+k}\right)^{R-1}\right] + \\ &\ldots + \left[\left(\frac{1+g}{1+k}\right)^{R-1}-\left(\frac{1+g}{1+k}\right)^{R-1}\right]\Bigg\} \end{aligned} \tag{A5}$$

Note that the last difference within the braces of (A5) equals 0, but that difference is retained in the expression for completeness.

Rearranging and combining the like terms within (A5) produces

$$\begin{aligned} &\alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\Bigg[-R\left(\frac{1+g}{1+k}\right)^{R-1} + \\ &1 + \left(\frac{1+g}{1+k}\right) + \left(\frac{1+g}{1+k}\right)^{2} + \ldots + \left(\frac{1+g}{1+k}\right)^{R-1}\Bigg] \end{aligned} \tag{A6}$$

Now, from within the brackets of (A6), consider the terms

$$1 + \left(\frac{1+g}{1+k}\right) + \left(\frac{1+g}{1+k}\right)^2 + \ldots + \left(\frac{1+g}{1+k}\right)^{R-1} \quad \text{(A7)}$$

The above is a series of the form $1 + a + \ldots + a^n$, where $a = (1+g)/(1+k)$ and $n = R - 1$. The sum of the above series equals $(a^{n+1} - 1)/(a - 1)$. Therefore, the sum of the terms of the series (A7) equals

$$\left[\left(\frac{1+g}{1+k}\right)^R - 1\right] \Big/ \left(\frac{1+g}{1+k} - 1\right) = \left[1 - \left(\frac{1+g}{1+k}\right)^R\right]\left(\frac{1+k}{k-g}\right) \quad \text{(A8)}$$

Substituting (A8) for (A7) into (A6) yields

PV of insurance costs over the first R years equals

$$\alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\left\{-R\left(\frac{1+g}{1+k}\right)^{R-1} + \left[1 - \left(\frac{1+g}{1+k}\right)^R\right]\left(\frac{1+k}{k-g}\right)\right\} \quad \text{(A9)}$$

Let $F(R)$ equal the expression in braces of (A9); that is,

$$F(R) = \left\{-R\left(\frac{1+g}{1+k}\right)^{R-1} + \left[1 - \left(\frac{1+g}{1+k}\right)^R\right]\left(\frac{1+k}{k-g}\right)\right\} \quad \text{(A10)}$$

Therefore, in terms of $F(R)$,

PV of insurance costs over the first R years

$$= \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)F(R) \quad \text{(A11)}$$

Now, consider the present value of the insurance costs over the next L years (after the first R years), $R \le t \le R + L - 1$. That sum is

(Annual insurance cost with L years remaining, paid at $t = R$) /
$(1 + k)^R$ + (Annual insurance cost with L–1 years remaining,
paid at $t = R+1$) / $(1+k)^{R+1}$
+...
+ (Annual insurance cost with 1 year remaining, paid at
$t = R+L-1$) / $(1+k)^{R+L-1}$ (A12)

The representation for the present value of these costs is similar to the representation for the first R years, as given in (A11), except for the following: $F(R)$ is replaced with $F(L)$; each discounted cash flow is multiplied by $(1+g)^R$ to account for the price increases over the first R years; and each cash flow is further discounted by $(1+k)^R$ to discount R additional years back to period 0. Consequently,

PV of the insurance costs, for $R \le t \le R + L - 1$,

$$= \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)F(L)\frac{(1+g)^R}{(1+g)^R} \tag{A13}$$

$$= \alpha C\left(\frac{1+g}{1+k}\right)^{R+1}\left(\frac{1+kk}{kk-gg}\right)F(L)$$

Similarly,

PV of insurance costs, for the next L years, $R + L \le t \le R + 2L - 1$,

$$= \alpha C\left(\frac{1+g}{1+k}\right)^{R+L+1}\left(\frac{1+kk}{kk-gg}\right)F(L) \tag{A14}$$

and so forth. Therefore, the present value of insurance costs over an infinite time period under the capital budgeting method, assuming no losses, is

$$\text{AGGN(CB)} = \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)F(R) + \alpha C\left(\frac{1+g}{1+k}\right)^{R+1} \tag{A15}$$

$$\left(\frac{1+kk}{kk-gg}\right)F(L) + \alpha C\left(\frac{1+g}{1+k}\right)^{R+L+1}\left(\frac{1+kk}{kk-gg}\right)F(L) + \ldots$$

and after factoring, the result is

$$\text{AGGN(CB)} = \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\Bigg\{F(R) + \tag{A16}$$

$$\left(\frac{1+g}{1+k}\right)^{R}\left[1 + \left(\frac{1+g}{1+k}\right)^{L} + \left(\frac{1+g}{1+k}\right)^{2L} + \ldots\right]F(L)\Bigg\}$$

Using the relationships established in (3) and (4), where $(1 + g)^L - 1 = gg$, and $(1 + k)^L - 1 = kk$, (A16) can be rewritten as

$$\text{AGGN(CB)} = \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right) \tag{A17}$$

$$\left\{F(R) + \left(\frac{1+g}{1+k}\right)^R\left[1 + \left(\frac{1+gg}{1+kk}\right) + \left(\frac{1+gg}{1+kk}\right)^2 + \ldots\right]F(L)\right\}$$

The terms in the brackets of (A17) equal 1 plus a standard growth model, and therefore those terms can be replaced by $1 + (1 + gg) / (kk - gg) = (1 + kk) / (kk - gg)$, and, therefore,

$$\text{AGGN(CB)} = \alpha C\left(\frac{1+g}{1+k}\right)\left(\frac{1+kk}{kk-gg}\right)\left[F(R) + \left(\frac{1+g}{1+k}\right)^R\left(\frac{1+kk}{kk-gg}\right)F(L)\right] \tag{A18}$$

This result was previously presented in the main part of the article.

Note that (A18) above represents the aggregate insurance costs over an infinite time period. If the aggregate cost over a finite time is desired, the intermediate result (A16) can be used. For example, assume that the remaining life of an asset is $R = 3$ and the normal life is $L = 10$. Then, the aggregate insurance costs over a 23-year period ($R + 2L$) would be calculated from (A16) but with just the first two terms of the expression in the brackets.

APPENDIX II

Derivation of Aggregate Insurance Costs under the Actual Cash Value Method (No Losses)

In this appendix, the aggregate cost of insurance is derived under the ACV method of evaluating exposures when the depreciation percentage is constant and when there are no losses. The aggregate cost of insurance, AGGN(ACV), is the sum of the annual insurance costs over an infinite time horizon, and the derivation parallels the derivation under the capital budgeting method, found in Appendix I.

The derivation begins by considering the sum of discounted insurance costs over the first R years. Using the annual insurance costs from (14), and representing these costs in terms of the current values of R and C, PV of insurance costs, over the first R years,

$$= \alpha C(1+g)(R-1)/L + \frac{\alpha C(1+g)^2(R-2)/L}{1+k} + \ldots + \frac{\alpha C(1+g)^{R-1}[R-(R-1)]/L}{(1+k)^{R-2}} + \frac{\alpha C(1+g)^R(R-R)/L}{(1+k)^{R-1}} \quad \text{(B1)}$$

After factoring,

PV of insurance costs, over the first R years,

$$= \alpha C(1+g)/L\left[(R-1) + (R-2)\left(\frac{1+g}{1+k}\right) + \ldots + 1\left(\frac{1+g}{1+k}\right)^{R-2} + 0\left(\frac{1+g}{1+k}\right)^{R-1}\right] \quad \text{(B2)}$$

Note that the last terms of both (B1) and (B2) equal 0; however, these terms are retained for completeness.

Consider the terms within the brackets of (B2). Those terms are of the form

$$(R-1) + (R-2)h + (R-3)h^2 + \ldots + [R-(R-1)]h^{R-2} + (R-R)h^{R-1}$$
$$= (R-1) + (R-2)h + (R-3)h^2 + \ldots + h^{R-2},$$

where $h = (1+g)/(1+k)$. The sum of the terms is[3] $\dfrac{h^R + R(1-h) - 1}{(1-h)^2}$ (B3)

Consequently, from (B2) and (B3)

PV of insurance costs, over the first R years,

$$= \alpha C(1+g)/L\left[\left(\frac{1+g}{1+k}\right)^R + R\left(1-\frac{1+g}{1+k}\right) - 1\right]\Big/\left(1-\frac{1+g}{1+k}\right)^2$$

$$= \alpha C(1+g)/L\left[\left(\frac{1+g}{1+k}\right)^R + R\left(\frac{k-g}{1+k}\right) - 1\right]\Big/\left(\frac{1+k}{k-g}\right)^2$$

$$= \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2}\left[\left(\frac{1+g}{1+k}\right)^R + R\left(\frac{k-g}{1+k}\right) - 1\right] \quad \text{(B4)}$$

$$\text{Let } H(R)\left[\left(\frac{1+g}{1+k}\right)^R + R\left(\frac{k-g}{1+k}\right) - 1\right] \tag{B5}$$

Then, in terms of $H(R)$,

PV of insurance costs, over R years,

$$= \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2} H(R) \tag{B6}$$

Now, consider the present value of insurance costs over the next L years (after the first R years). The representation for that cost is the same as (B6), except for the following: R is replaced with L; each cash flow is multiplied by $(1 + g)^R$ to account for price increases over the first R years; and each cash flow is further discounted by $(1 + k)^R$ to discount R additional years back to 0. Consequently,

PV of insurance costs for the years $R + L$ to $R + L - 1$

$$= \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2} \frac{(1+g)^R}{(1+k)^R} H(L) \tag{B7}$$

Similarly, as was demonstrated in Appendix I,

PV of insurance costs, for the next L years, $R + L \le t \le R + 2L - 1$,

$$= \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2} \frac{(1+g)^{R+L}}{(1+k)^{R+L}} H(L) \tag{B8}$$

and so forth. Therefore, the present value of insurance costs over an infinite time period under the ACV method, assuming no losses, is

$$\begin{aligned} \text{AGGN(ACV)} = {} & \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2} H(R) + \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2} \frac{(1+g)^R}{(1+k)^R} H(L) \\ & + \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2} \frac{(1+g)^{R+L}}{(1+k)^{R+L}} H(L) + \\ & \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2} \frac{(1+g)^{R+2L}}{(1+k)^{R+2L}} H(L) + \ldots \end{aligned} \tag{B9}$$

and after factoring and some simplifying, the result is

$$\text{AGGN(ACV)} = \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2}\left\{H(R) + \frac{(1+g)^R}{(1+k)^R}\left[1 + \frac{(1+g)^L}{(1+k)^L} + \frac{(1+g)^{2L}}{(1+k)^{2L}} + \ldots\right]H(L)\right\} \tag{B10}$$

Using the relationships established in (3) and (4), where $(1+g)^L - 1 = gg$, and $(1+k)^L - 1 = kk$, (B10) can be rewritten as

$$\text{AGGN(ACV)} = \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2}\left[H(R) + \frac{(1+g)^R}{(1+k)^R}\left(1 + \frac{(1+gg)^2}{(1+kk)^2} + \ldots\right)H(L)\right] \tag{B11}$$

The terms in the brackets of (B11) equal 1 plus a standard growth model, and therefore, those terms can be replaced by $1 + (1 + gg)/(kk - gg) = (1 + kk)/(kk - gg)$; therefore,

$$\text{AGGN(ACV)} = \frac{\alpha C(1+g)(k-g)^2}{L(1+k)^2}\left[H(R) + \frac{(1+g)^R}{(1+k)^R}\left(\frac{1+kk}{kk-gg}\right)H(L)\right] \tag{B12}$$

This result was previously presented in the main part of the article.

Note that (B12) above represents the aggregate insurance costs over an infinite time period. If the aggregate cost over a finite time is desired, the intermediate result (B10) can be used. For example, assume that the remaining life of an asset is $R = 3$, and the normal life is $L = 10$. Then the aggregate insurance costs over a 23-year period ($R + 2L$) would be calculated from (B10) but with just the first two terms of the expression in the brackets.

ENDNOTES

[1] As is the case with constant-growth stock valuation models, this model requires the assumption that the firm's cost of capital be greater than the rate of increase in asset prices. This

assumption is reasonable because required rates of return over the long run are generally assumed to include a real rate of return and risk premiums in addition to the premium for expected inflation.

[2] Assuming a constant rate of wear and tear, as opposed to an accelerated rate, will result in more conservative (higher) estimates of the actual cash value in the early years of an asset's life.

[3] Let $S = (R-1) + (R-2)h + (R-3)h^2 + \ldots + (R-(R-1))h^{R-2} + (R-R)h^{R-1}$. Then, $hS = (R-1)h + (R-2)h^2 + \ldots + (R-(R-1))h^{R-1} + (R-R)h^R$, and $S - hS = S(1-h) = (R-1) - (h + h^2 + \ldots + h^{R-1})$. The second set of terms is a geometric progression and equals $(h^R - h)/(h-1)$. Substituting this value for the infinite sum yields
$S(1-h) = (R-1) - [(h-h^R)/(1-h)]$. After some simplification, $S = [h^R + R(1-h) - 1]/(1-h)^2$.

REFERENCES

Doherty, Neil A. (1985) *Corporate Risk Management.* New York: McGraw-Hill, pp. 26–27.

Head, George L., and Stephen Horn II (1985) *Essentials of the Risk Management Process,* Volume I. Insurance Institute of America, pp. 133–148.

Huebner, S.S., Kenneth Black Jr., and Robert S. Cline (1982) *Property and Liability Insurance,* 3rd. ed. Englewood Cliffs, NJ: Prentice-Hall Inc.

Mehr, Robert I., and Bob A. Hedges (1974) *Risk Management: Concepts and Applications.* Homewood, IL: Richard D. Irwin, Inc., pp. 491–492.

Rodda, William H., James S. Trieschmann, Eric A. Wiening, and Bob A. Hedges (1988) *Commercial Property Risk Management and Insurance.* American Institute for Property and Liabilty Underwriters Inc., Malvern, PA.

Vaughn, Emmett J. (1989) *Fundamentals of Risk and Insurance.* New York: John Wiley and Sons Inc.

Williams, C. Arthur Jr., Michael L. Smith, and Peter C. Young (1995) *Risk Management and Insurance,* 7th ed. New York: McGraw-Hill Inc.