The Impact of Cash Flow Volatility on Systematic Risk

Nicos A. Scordis, * James Barrese,** and Ping Wang***

Abstract: In a word, where information is costly, volatile cash flows create information acquisition costs that reduce value. Thus, managers act to reduce their firm’s volatility of cash flow in anticipation of higher value for shareholders. However, when managers reduce the firm’s cash flow volatility, they also affect the systematic risk of their firm’s stock. The direction of the relationship between cash flow volatility and systematic risk depends on the relative value of the firm’s growth opportunities in relation to the firm’s assets-in-place. We use a panel sample of 542 observations from United States insurance firms to investigate the relationship between cash flow volatility and systematic risk. The direction of the relationship between cash flow volatility and systematic risk has implications both for the education and for the practice of risk management. We make recommendations for risk management programs. Our findings also have implications for clienteles among the firm’s stockholders. While the theoretical relationship between cash flow volatility and systematic risk can be generally applied, we only test this relationship for a sample of insurance firms. [Key words: Cash flow volatility, beta, total risk, systematic risk]

INTRODUCTION

The pointed-hair boss in the Dilbert cartoon strip (United Feature Syndicates, August 8, 1992) refuses a pay-raise to a subordinate. The boss’s reasoning is that the raise would induce volatility in the firm’s cash flow, which would upset all the stock market analysts, particularly the lazy ones. Theory also advocates smooth cash flows (Froot, Scharfstein, and Stein, 1993; Smith and Stulz, 1985). At times, cash flows with high volatility...
result in insufficient cash to fund profitable projects. Sometimes the projects are so compelling that managers can easily raise funds. Often, however, they are not. The result is a pattern of underinvestment which reduces expected cash flow. In fact, Adam (2002) shows that firms chose their volatility-management strategies according to the cost of raising external funds. In a world where information is costly, volatile cash flow exacerbates information asymmetries as Goel and Thakor (2003) show. Outsiders need information in sufficient clarity to calculate the true value of the firm. Otherwise they tend to undervalue the firm. Thus, managers act to reduce their firm’s volatility of cash flow in anticipation of higher shareholder value. However, when managers reduce their firm’s cash flow volatility, they also affect the systematic risk of their firm’s stock. The direction of the relationship between cash flow volatility and systematic risk depends on the relative value of the firm’s growth opportunities. This study empirically investigates this relationship.

The direction of the relationship between cash flow volatility and systematic risk has implications for both the education and practice of risk management. One way shareholders value their stock is to estimate the size of future cash flows and use an appropriate rate for a discounting of each future cash flow. The risk that stockholders cannot diversify away—the systematic risk of the stock—helps determine the discounting rate. This rate is the stockholder’s cost of capital. Influential textbooks on risk and insurance explain that reduced volatility in a firm’s cash flow increases the size of expected cash flows while leaving the stockholder’s cost of capital unchanged (Harrington and Niehaus, 2003, Chapter 20; Williams, Smith, and Young, 1998, Chapter 3). In general, we agree with the textbooks. Reductions in total risk should leave the shareholder’s cost of capital unchanged because risk reductions by the firm’s managers reduce the type of risk that shareholders themselves can eliminate by holding diversified portfolios; Sharpe’s famous Capital Asset Pricing Model (CAPM) is built on this idea (Sharpe, 1964). If, however, we recognize the limited liability nature of stock, the stockholders have an incentive to increase risk. If the higher risk pays off, the stockholders enjoy windfall profits. If the risk does not pay off, immunity from liability protects the stockholders from losing more than their investment. Thus, risk reductions by the firm’s managers could increase the relative risk of shareholders because shareholders give up possible windfall profits, without a corresponding increase in their immunity from liability.

Ultimately, of course, we need to understand the effect of risk reductions on the value of a firm. The prevailing view is that reducing cash volatility increases a firm’s expected cash flow, thereby increasing its value. Alternatively, the survey by Smithson and Simkins (2005) of the empirical
evidence suggests that the cash flow benefits of risk reduction might be
less than imagined. If this is true, the relationship between the volatility of
cash flow and systematic risk gains importance. A positive relationship
suggests that risk reduction may still create value even if it does not
increase the firm’s expected cash flow. A negative relationship, however,
suggests that if risk reduction is to create value, it must generate cash flows
greater than those needed to compensate for increased systematic risk.

**REVIEW OF THE LITERATURE**

**Underinvestment, Information Asymmetry, Cash Flow Volatility, and Value**

Cash flows with high volatility result in a pattern of underinvestment
that reduces value. Volatile cash flows also exacerbate information asym-
metries. Information asymmetries and high information acquisition costs
reduce firm value because outsiders need sufficient clarity in order to
calculate the true value of the firm. Otherwise, they tend to undervalue the
firm. Available empirical evidence is able to link cash flow volatility with
investment opportunities and information asymmetry. The empirical evi-
dence, however, that links cash flow volatility and firm value is mixed.

Schrand and Unal (1998) explain that core risks (those that stem from
activities for which a firm has some informational advantage) create eco-
nomic value while non-core risks do not.\(^3\) They continue to argue that a
firm should pay others to take on its non-core risk, thus enabling the firm
to take on even more core risk. Taking on core risk in place of non-core risk
reduces information asymmetry and creates shareholder value. This view
has influential supporters in the practice of corporate risk management.

Trevor Harris, a participant in the Morgan Stanley Roundtable on Risk
Management (2005) views risk management as being “about thinking
through ... information asymmetries and determining who has the greatest
comparative advantage in managing a given risk” (page 38). Harry Koppel,
another participant, explains that his firm’s scale and scope of “operations,
combined with years of operating experience, put[s] him in a better posi-
tion than any insurance company to evaluate, price and bear those risks”
(page 40). By allocating risk according to who is best able to manage it,
Charles Smithson and Tom Copeland, both participants in the roundtable,
agree that risk management helps the “company carry out its investment
policy, its strategic plan ... to ensure a company’s ability to carry out its
business plan” (page 36). John Kapitan, also a participant, clarifies that if
there is a lot of noise in the firm’s performance, investors do not know if
managers “are good operators, or just lucky and riding favorable commod-
ity price movements…. By hedging, and thus removing the effect of [commodity] price volatility on its cash flows … a company effectively ends up with a much less noisy performance measure” (page 52).

As these risk managers suggest, paying others to take on a firm’s non-core risk reduces information asymmetry between managers and investors. Asymmetric information arguments fall into agency and signaling models. The theme of agency models is that investors observe only part of the actions of managers and are thus unable to enforce contracts in which success ultimately depends on managerial effort. Since the actions of managers affect the wealth of investors, managers need to bond their actions if they are to entice investors to finance new projects. Purchasing insurance or derivatives on the project’s output is one way for managers to bond their actions (Campbell and Kracaw, 1987; MacMinn, 1987; Mayers and Smith, 1987). Signaling models deal with the comparatively superior information managers have compared to the information outsiders (such as shareholders or analysts) have on the firm’s cash flow. An observable decision to lock-in some aspect of the firm’s cash flow conveys information from which outsiders can infer the firm’s true value (DeMarzo and Duffie, 1995; Zhao, 2004).

Minton, Schrand, and Walther (2002) find that when cash flows are volatile, instead of incorporating such volatility in their stock price forecasts, outsiders undervalue the firm. Géczy, Minton, and Schrand (1997) as well as Minton and Schrand (1999) find that higher cash flow volatility is associated with lower investment. However, while Carter, Rogers, and Simkins (2006), as well as Allayannis and Weston (2001), conclude that reducing cash volatility increases value, Jin and Jorion (2006) as well as Tufano (1996) conclude that reducing cash volatility does not increase value. The survey by Smithson and Simkins (2005) documents further such mixed empirical results. For example, some of the studies surveyed show a positive correlation between share values and the use of foreign exchange and interest rate derivatives, while others show no such correlation. The lack of empirical consensus is not surprising given the patchwork of data these studies represent. Data availability forces empirical studies on managing risk to limit their examination to a single risk financing technique (such as derivatives or insurance), to analyze individual firms (such as Delta Airlines or Cephalon Inc.), or to rely on single industry samples (such as the oil industry, gold-mining industry, or insurance industry). Therefore, it is difficult to compare results and generalize across empirical studies. In fact, it can be argued that while there is a general theory of managing risk, the structures resulting from the application of this theory are industry specific. The implications of the risk financing models of Boyle and Guthrie
The model of Adam (2002) points to differences in firms’ credit risk premium (i.e., the difference between the cost of internal funds and the cost of raising external funds) to explain the observed diversity in the ways firms finance the consequence of risk. Even within a single industry, firms with the smallest credit risk premium aim at preserving cash flow for future projects. These firms implement convex risk financing strategies (such as purchasing put options) to generate additional cash flow against times of expected cash shortfall. Firms with the largest credit risk premium use a different tactic, aiming instead to preserve cash flow for current projects. They implement concave risk financing strategies (such as selling call options) to ensure the size of their cash flow stream. In this model, when shareholders lock in cash for projects they implicitly surrender valuable rights to choose among future alternatives. As the value of the surrendered alternative increases, the convexity or concavity of the risk financing strategy also increases.

The right of choice is also central in the model of Boyle and Guthrie (2003). In this model, shareholders can delay or later scale up a project. In the event of cash shortfall, however, shareholders face uncertainty about the future payoff of the project itself as well as uncertainty about their ability to finance later the completion of the project. According to Boyle and Guthrie (2003), these two sources of uncertainty have opposing effects on the relationship between cash flow volatility and investment opportunities, and thus firm value. Since volatility measurements contain elements of both payoff and financing uncertainty, their opposing effects can help explain why it is difficult to obtain an empirical consensus.

How Systematic Risk Changes

Callahan and Mohr (1989) provide a comprehensive synthesis of the research on the determinants of systematic risk. They conclude that firm size, financial leverage, and future growth often appear as central determinants of a firm’s stock beta. Many researchers assume a negative relationship between a firm’s size and its stock beta under the assumption that a larger firm creates cash flows invariant to the economy. Bowman (1979, p. 626), however, explains that since the size of a firm “is in no way dependent upon the capital structure of the firm … there is no necessary theoretical relationship between size and systematic risk.” Indeed, Daves et al. (2000), who examine the relationship between systematic risk and firm size over a 34-year period, conclude that the empirical relationship between size and beta is driven by changes in the financial leverage of firms as the size of firms increases. They find both a negative and a positive relationship
between size and beta. After 1980 they find that large firms exhibit a larger beta because larger firms use more debt in their capital structures.

The rates we all use to estimate a firm’s beta include both operating and financial risk. Thus, the estimated beta also includes both types of risk. This empirical beta is the leveraged beta ($\beta^L$) of the firm or the stock’s beta ($\beta_s$). By comparison, the unleveraged beta ($\beta^U$) represents the assumed systematic risk of the firm if the firm is financed entirely with shares of equity (E), and no debt (D). Hamada (1969) and Conine (1980) derive a relationship between the observed leveraged beta and the firm’s unleveraged beta. Conine (1980) shows that $\beta^L = \beta^U[1 + (1 - \text{Tax}_{\text{Corporate}})D/E] - \beta_{\text{Debt}}[(1 - \text{Tax}_{\text{Corporate}})D/E]$. When the firm’s debt is assumed to be free from systematic risk ($\beta_{\text{Debt}} = 0$) then the Conine (1980) formulation reduces to the popular Hamada (1969) formulation of $\beta^L = \beta^U[1 + (1 - \text{Tax}_{\text{Corporate}})D/E]$ and with $\partial \beta^U / \partial D > 0$. Logically, this is consistent with the idea that for the stockholders who are residual claimants, leveraged investments are riskier than unleveraged ones, or alternatively that the opportunity cost of capital to shareholders increases with increases in the market value ratio of debt to equity.

A firm can achieve growth in two ways. One way is to take on new projects with expected return and commensurate risk higher than that of its existing projects. This type of growth, as Bowman (1979) points out, is comparable to increasing the firm’s size and, thus, it does not influence the firm’s beta. Another way to grow is for the firm to take on projects promising excess return. This is the view of growth opportunities adopted in Miller and Modigliani (1961) as well as in this and other papers. In a competitive economy, excess return, economic profit, or positive net present value spring from a competitive advantage the firm possesses. Over time, as Oster (1999) explains, market forces erode a firm’s competitive advantage, so projects that used to earn the firm excess return no longer do so. Senbet and Thompson (1982), Miles (1986), and Chung and Charoenwong (1991) provide theoretical and empirical evidence in favor of a positive relationship between such profitable future opportunities and the firm’s stock beta.

Indeed, Miller and Modigliani (1961) explain that there are two drivers in the value of the firm’s equity. These are the market value of cash flows generated by the firm’s assets-in-place and the present value of the firm’s future growth opportunities. Miles (1986) explains that systematic risk is a value-weighted average of assets-in-place and the growth options embedded in these assets. Thus, the beta of stock ($\beta_s = W_A \beta_A + W_O \beta_O$) is the value-weighted average of the systematic risk of cash flows from the firm’s assets already in place ($\beta_A$) and of the systematic risk of cash flows from the growth options ($\beta_O$) embedded in these assets. The weights $W_A = A/(A+O)$
and $W_O = O/(A+O)$ are the fraction of value attributable to assets ($A$) and to their embedded options ($O$), respectively.

When managers reduce the volatility of the cash flow generated by the firm’s assets ($A$) they create two competing effects. First, the systematic risk of assets ($\beta_A$) decreases while the systematic risk of the options ($\beta_O$) increases (we show this in Appendix A). Second, the value of the growth options embedded in the firm’s assets ($O$) declines while the value of assets already in place ($A$) increases. Therefore, a reduction in the volatility of cash flow creates two pairs of competing effects. For a firm with large asset value in relation to its future opportunities, the net effect of a reduction in the volatility of cash flow will be a decrease in the firm’s stock beta. In contrast, when a firm has small asset value in relation to its future opportunities, the net effect of a reduction in the volatility of cash flow will be an increase in the firm’s stock beta. These results flow from the contingent claim nature of equity. Higher total risk increases the stockholders’ chance of windfall profits, while the limited liability feature of their stock protects them from losing more than their investment.

There is disagreement about how best to estimate the stockholder’s cost of capital. All agree, however, that some of the drivers of the cost of capital are macroeconomic. The CAPM collapses all macroeconomic drivers into a single factor: the required return on the market portfolio. The sensitivity of stock risk to changes in the market portfolio, or the stock’s beta, is determined by factors like the firm’s size, market power, prospects for future growth, and leverage. Beta, therefore, measures the marginal contribution of the stock to the risk of the market portfolio. If all investors hold the market portfolio, then the risk premium investors require, and accordingly the cost of capital, is directly proportional to beta. Rather than navigate the disagreement over what other factors in addition to beta may be needed to accurately estimate the stockholder’s cost of capital, we instead estimate the stock’s beta.

No studies that examine the direction of the relationship between the total risk of an insurer’s cash flow and the beta of the firm’s stock currently exist. Results from non-insurer studies only indirectly pertain to the direction of this relationship. Minton and Schrand (1999) conclude that there is no relationship between a firm’s cash flow risk and beta, but attribute this finding to biases in estimation. The studies of Guay (1999), Hentschel and Kothari (2001), and Lin (2003), which compare the stock beta of firms that use derivatives and those that do not use derivatives, are similar in empirical design and include overlapping samples. Hentschel and Kothari (2001, Table 2) and Lin (2003, Table 4) find that, for firms using derivatives, stocks have a higher average beta than the stocks of firms that do not use derivatives. While this result is suggestive of a positive relationship
between total risk and beta, Guay’s results (1999, Table 2) do not confirm such a relationship.

RESEARCH DESIGN

Panel Data

In addition to total cash flow risk, we explicitly measure firm size, future growth, and financial leverage. We summarize in Table 1 our expected results given the theoretical relationships supporting our investigation. We also establish a control for omitted variables by using panel data. Panel data contain information both across firms and over time for each firm. Using dummy variables to represent individual firms in our sample, we control for omitted variables by assuming that the firm-specific dummy variables may be either fixed or random. We use 542 cross-sectional, time-series observations of US insurance firms with North American Industry Classification System (NAICS) code 524126. This code identifies direct property and casualty insurance carriers. Our observations are from 54 firms over the data years 1991 to 2004. Our selection of these firms was based on a process of elimination that began with all 161 of the property and casualty insurance firms filing Form 10-K with the Securities and Exchange Commission (SEC) for the year 2004. We report the firms in our sample and their data years in Appendix B.

Dependent Variable

We denote annual beta, our dependent variable, as (Beta). We estimate a Scholes-Williams beta using daily closing stock return data in conjunction with a value-weighted market return. We use daily data because, for each firm in our panel of data, we need to estimate a beta for each year. Only daily data provide the large number of observations necessary to estimate accurately these annual betas. If we use the traditional ordinary-least-squares estimation method of beta, the use of daily data results in estimation errors. For this reason, we estimate beta according to the method developed by Scholes and Williams (1977). In their comparative study of beta estimation methods, McInish and Wood (1986) favor this method, especially when daily share trading is not high. The median, mean, and standard deviation values of (Beta) are 0.623, 0.673, and 0.414, respectively.

Independent Variables

We interpret and measure total cash flow risk as the variability of operating cash flow. To estimate this variability we first establish a trend
line using the quarterly operating cash flow for each firm in the sample. We do this by fitting a regression line $C(t) = \alpha_0 + \alpha_1(t)$ where $C(t)$ is each of the quarterly cash flows and $(t)$ is the time to obtain a smoothed estimate $\hat{C}(t)$. We then measure the absolute deviation between each quarterly cash flow observation from this trend, $C(t) - \hat{C}(t)$. Lastly, we calculate the average of each year’s quarterly deviations from the trend

$$\frac{1}{4} \sum_{t=1}^{4} |C(t) - \hat{C}(t)|$$

This quarterly average deviation from the trend is our measurement of cash flow risk for each year. We denote operating cash flow risk as (OCF Risk). The median, mean, and standard deviation values of (OCF Risk) are $47$ million, $168$ million, and $365$ million, respectively.

A control for the total risk of cash flow from investing and financing activities is also established. We include the total cash flow risk from investing and financing activities as controls because these factors may become more important than cash flow from operations as a firm moves through the stages of start-up, growth, maturity, and decline (Black, 1998; Livnat and Zarowin, 1990). The median, mean, and standard deviation values of the total cash flow risk from investments as (CFI Risk) are $65$ million, $236$ million, and $280$ million, respectively. The median, mean and standard deviation values of the total risk of cash flow from financing (CFF Risk) are $27$ million, $122$ million, and $280$ million, respectively.

We determine the current size of the firm’s assets by measuring the market value of assets. The firms in our sample report the market value of their assets either on their balance sheet, in parenthetical references within the balance sheet, or in notes included in the financial statements. The great majority of these assets are financial. The remaining assets are physical assets consisting mostly of real estate held for investment and infrastructure. The median, mean, and standard deviation values of the market value of assets (Asset Value) are $1,985$ million, $9,627$ million, and $19,291$ million, respectively.

We measure the size of the firm’s future opportunities as the insurer’s stock market value divided by the stock’s book value. It is not possible to determine the value of a firm’s options but, as an aid to analysis, the market-to-book ratio is commonly used for this purpose (see, for example, Smith and Watts, 1992; Collins, Blackwell, and Sinkey, 1994; or Graham and Rogers, 2002). The stock’s book value represents the insurer’s equity value that would remain if the insurer were liquidated and assets and liabilities are accurately represented on the balance sheet. Firms are generally expected to grow in order to generate sustainable economic profit for their stockholders. When stockholders believe in the ability of managers to
Table 1. Expected Results

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Expected Result</th>
<th>Basis for expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Risk</td>
<td>+ (−)</td>
<td>When asset value is large (small) in relation to future growth opportunities, a decrease in total risk results in a decrease (increase) in stock beta. We measure total risk as variable (OCF Risk) as well as (CFI Risk) and (CFF Risk). Miles (1986) expresses stock beta ($\beta_S$) as a value-weighted average of the systematic risk of cash flows from the firm’s asset already in place ($\beta_A$) and of the systematic risk of cash flows from growth options ($\beta_O$) embedded in these assets, or $\beta_S = \frac{A}{A+O}\beta_A + \frac{O}{(A+O)}\beta_O$. It is generally accepted that as total risk decreases, the value of the assets (A) increases while the value of their associated options (O) decreases. In Appendix A we show that as total risk decreases, ($\beta_A$) decreases but ($\beta_O$) increases.</td>
</tr>
<tr>
<td>Current Size</td>
<td>+</td>
<td>A large firm has a relatively large beta because it uses more debt in its capital structures. We measure size as variable (Asset Value). While Bowman (1979) explains that the theoretical relationship between a firm’s size and its stock beta is vague, empirical evidence shows both a positive and negative relationship between size and beta. Daves et al. (2000) find that is due to changes in the financial leverage of firms.</td>
</tr>
<tr>
<td>Future Growth</td>
<td>+</td>
<td>A firm that takes on projects that promise excess return has a larger stock beta because a larger proportion of its value is derived from future but as yet unrealized cash flows. We measure future opportunities as variable (Future Growth). Senbet and Thompson (1982), Miles (1986), and Chung and Charoenwong (1991) document a positive relationship between future growth and the firm’s stock beta.</td>
</tr>
<tr>
<td>Leverage</td>
<td>+</td>
<td>Stockholders are residual claimants, and thus for them a leveraged firm is riskier than an unleveraged firm. We measure leverage as variables (Bh Debt) and (Ph Debt). The firm’s observed stock beta ($\beta_S \equiv \beta^L$) reflects the firm’s actual capital structure. By comparison, the unleveraged beta ($\beta^U$) represents an assumed capital structure of only equity (E) and no debt (D). Even if the firm’s debt has no systematic risk, Hamada (1969) explains that $\beta^L = \beta^U[1 + (1 - \text{Tax}_{\text{Corporate}})D/E]$.</td>
</tr>
</tbody>
</table>
convert these growth opportunities into value (or, in the language of
derivatives, exercising the insurer’s in-the-money real options) they will
pay more than the residual value of assets for the firm’s stock. Accordingly,
the firm’s market-to-book ratio exceeds one. The median, mean, and stan-
dard deviation values of the market-to-book ratio—our representation of
the firm’s future growth opportunities (Future Growth)—are 1.588, 2.335
and 2.335, respectively.

According to the empirical findings of Cummins and Lamm-Tennant
(1995), an insurer’s financial leverage depends on two components. The
first component is funds the insurer receives from its bondholders. This is
the insurer’s financial debt. The relative size of financial debt is measured
as the face value of long-term debt divided by the stockholder’s equity. We
denote this ratio as (Bh Debt). The median, mean, and standard deviation
values of (Bh Debt) are 0.159, 0.204, and 0.231, respectively. The second
component is funds the insurer receives from its policyholders. This is the
firm’s policyholder debt. The relative size of policyholder debt is calculated
as the face value of gross reserves divided by stockholder’s equity. We
denote this ratio as (Ph Debt). The median, mean, and standard deviation
values of (Ph Debt) are 1.558, 1.694, and 1.201, respectively.

RESULTS

In drawing inferences from panel data we can investigate the economic
relationships captured by the data under different analysis assumptions.
We can assume that the effect of omitted variables captured by the dummy
variables is either fixed or random. We present analysis results under both
of these assumptions. When we assume fixed effects we use a least squares
dummy variable procedure. When we assume random effects we use a
feasible generalized least squares procedure. The Hausman (1978) test,
however, favors the fixed effects assumption.

The purpose of our analysis is to investigate the relationship between
total cash flow risk and beta. Our literature review suggests that this is not
a direct causal relationship. It is instead a moderated causal relationship.
The nature of the relationship between total cash flow risk and beta varies
depending on the value of current assets and the prospect for growth. We
represent this moderating relationship with multiplicative interaction
terms (OCF Risk \times Asset Value) and (OCF Risk \times Future Growth). We do
the same for (CFI Risk) and (CFF Risk).

Table 2 contains the results from the random effects estimation and
Table 3 contains the results from the fixed effects estimation. It should be
noted that, for the sake of readability, we do not report the estimated
coefficients of the dummy variables in Table 2 or in any subsequent fixed effects table. The conditional coefficients represent the adjustment of estimated coefficients due to the interaction of variables, while the conditional p-value represents the statistical significance of the conditional coefficient. Conditional elasticity is estimated at mean sample values; the term conditional here indicates coefficients and statistical significance levels adjusted for the presence of the multiplicative interaction terms. Consider, for example, the following regression relationship:

\[
(Beta)_{it} = \gamma_1 (OCF Risk)_{it} + \gamma_2 (OCF Risk \times Asset Value)_{it} + \gamma_3 (OCF Risk \times Future Growth)_{it} + \gamma_4 (Asset Value)_{it} + \gamma_5 (Future Growth)_{it} + \gamma_6 (Bh Debt)_{it} + \gamma_7 (Ph Debt)_{it} + error
\]

where subscripts \(i\) and \(t\) define an insurer and a year. The estimated coefficient of variable (OCF Risk) is the main effect while the estimated coefficients of each of the interaction variables are the moderated effects. The sign of the main effect tells us the direction of the average effect of (OCF Risk) on (Beta). The respective signs of the moderated effects tell us how each of the moderating variables (Asset Value) and (Future Growth) influences the main effect. The conditional estimated coefficient of (OCF Risk) tells us how different values of (Asset Value) and (Future Growth) influence the main relationship between (OCF Risk) and (Beta). The conditional estimated coefficient is:

\[
\bar{\gamma}_1 = \frac{\partial(Beta)}{\partial(OCF \ Risk)} = \hat{\gamma}(1) + \hat{\gamma}(4)(Asset \ Value) + \hat{\gamma}(4)(Future \ Growth)
\]

(2)

and the conditional standard error of coefficient \(\hat{\gamma}_1\) is:

\[
\hat{s}_{\hat{\gamma}(1)} = \left\{ \sum_{k=1}^{N} x_{\gamma(k)}^2 s_{\gamma(k)}^2 + \sum_{k=1}^{N} \sum_{\lambda=1}^{N} x_{\gamma(k)} x_{\gamma(\lambda)} Cov(s_{\gamma(k)}, s_{\gamma(\lambda)}) \right\}^{1/2}
\]

(3)

where \(\hat{\gamma}(k)\) denotes the regression’s estimated coefficients, \((x)\) denotes their respective variable values, \(s_{\gamma(k)}\) denotes the estimated coefficients’ standard error, and Cov(.) is the covariance operator. The usual statistical significance test is carried out using the ratio \(\hat{\gamma}_1 / \hat{s}_{\hat{\gamma}_1}\) and critical t-distribu-
If the conditional estimated coefficient is significant, then \( \left( \frac{OCF \text{ Risk}}{\text{Beta}} \right) \) is the elasticity of (OCF Risk).

**Estimated Sign of (OCF Risk)**

In Tables 2 and 3, the estimated coefficients of (OCF Risk), (OCF Risk × Asset Value), and (OCF Risk × Future Growth) are statistically significant.
The negative sign of the main effects (OCF Risk) indicates, on average, a negative relationship between (OCF Risk) and (Beta). A decrease in the risk of the operating cash flow increases the stock’s beta. The sign of the

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Estimated coefficient</th>
<th>p-value</th>
<th>Conditional coefficient</th>
<th>Conditional p-value</th>
<th>Conditional Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCF Risk</td>
<td>−2.326×10⁻⁴</td>
<td>0.044</td>
<td>−7.952×10⁻⁵</td>
<td>0.263</td>
<td>−</td>
</tr>
<tr>
<td>OCF Risk × Asset Value</td>
<td>−3.847×10⁻⁸</td>
<td>0.059</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OCF Risk × Future Growth</td>
<td>8.137×10⁻⁵</td>
<td>0.079</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset Value</td>
<td>1.284×10⁻⁵</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Future Growth</td>
<td>−5.912×10⁻³</td>
<td>0.331</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bh Debt</td>
<td>6.971×10⁻²</td>
<td>0.279</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ph Debt</td>
<td>6.082×10⁻²</td>
<td>0.011</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFI Risk</td>
<td>−1.090×10⁻⁴</td>
<td>0.121</td>
<td>1.409×10⁻⁴</td>
<td>0.063</td>
<td>0.000</td>
</tr>
<tr>
<td>CFI Risk × Asset Value</td>
<td>−1.597×10⁻⁹</td>
<td>0.114</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFI Risk × Future Growth</td>
<td>6.034×10⁻⁵</td>
<td>0.067</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset Value</td>
<td>9.325×10⁻⁶</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Future Growth</td>
<td>−9.834×10⁻⁴</td>
<td>0.466</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bh Debt</td>
<td>−4.683×10⁻²</td>
<td>0.346</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ph Debt</td>
<td>6.525×10⁻²</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFF Risk</td>
<td>2.975×10⁻⁵</td>
<td>0.415</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFF Risk × Asset Value</td>
<td>−2.815×10⁻⁹</td>
<td>0.138</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFF Risk × Future Growth</td>
<td>4.472×10⁻⁵</td>
<td>0.154</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset Value</td>
<td>8.115×10⁻⁶</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Future Growth</td>
<td>8.607×10⁻⁴</td>
<td>0.459</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bh Debt</td>
<td>−9.043×10⁻²</td>
<td>0.275</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ph Debt</td>
<td>6.384×10⁻²</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The dependent variable is the beta of a firm’s shares*
interaction term (OCF Risk × Asset Value) is negative while the sign of the interaction term (OCF Risk × Future Growth) is positive. This tells us that (Asset Value) lessens the negative relationship between (OCF Risk) and (Beta) while (Future Growth) amplifies this negative relationship. Thus, these signs suggest that as assets increase we should observe a negative moving to a positive relationship between operating cash flow risk and the stock’s beta. Conversely, as future growth opportunities increase, we should observe a positive moving to a negative relationship between operating cash flow risk and the stock’s beta.

These results are consistent with our expectations from Table 1. When managers reduce total risk, they create two opposing effects. At the extreme, for firms with only assets and no future growth opportunities, risk reduction reduces beta; for firms with no assets and only future growth opportunities, risk reduction increases beta. The signs of the control variables for asset values, future growth opportunities, and leverage, when significant, are consistent with our expectations as we summarize them in Table 1. On average, these opposing effects can cancel each other out. We can see this by calculating the conditional coefficient of variable (OCF Risk).

The conditional coefficient represents the net effect of asset value and future growth opportunities on the relationship between operating cash flow risk and the stock’s beta. At the mean sample values for (Asset Value) and (Future Growth) the mitigating effect of assets and the amplifying effect of growth cancel out the relationship between operating cash flow risk and the stock’s beta. Consequently, the conditional coefficients of (OCF Risk), (CFI Risk), and (CFF Risk) are not significantly different from zero, or their impact on (Beta) is effectively zero.

Specifically, in Table 2, the main effect is negative for (OCF Risk) and (CFI Risk) with estimated coefficients for the main effects \(-2.326 \times 10^{-4}\) and \(-1.090 \times 10^{-4}\), respectively. The moderated effects are negative for assets; the estimated coefficients for (OCF Risk × Asset Value) and for (CFI Risk × Asset Value) are \(-3.847 \times 10^{-8}\) and \(-1.597 \times 10^{-9}\), respectively. The moderated effects are positive for growth opportunities; the estimated coefficients for (OCF Risk × Future Growth) and for (CFI Risk × Future Growth) are \(+8.137 \times 10^{-5}\) and \(+6.034 \times 10^{-5}\), respectively. The estimated conditional coefficient of (OCF Risk) is not significantly different from zero. The p-value for the estimated conditional coefficient of (CFI Risk) suggests a 93.7 percent probability that it is significantly different from zero at the average sample values of (Asset Value) and (Future Growth). The conditional elasticity, however, is 0.000. This suggests that at the average sample values of (Asset Value) and (Future Growth), change in (CFI Risk) is not associated with a change in (Beta). The Hausman (1978) test favors these fixed effects
results over the random effects results. Nevertheless, the results of Table 3 support an identical conclusion as these fixed effects results.

In addition to using average sample values to evaluate the conditional coefficient for (OCF Risk) and its elasticity, we also use a range of values. This allows us to gain an intuitive feel for all possible values the conditional elasticity of (OCF Risk) can take in the presence of the moderating effects (Asset Value) and (Future Growth). It is assumed that each estimated coefficient of the regression Relationship (1) from Table 2 is normally distributed with mean $\hat{\gamma}_1$, $\hat{\gamma}_4$, and $\hat{\gamma}_5$ and standard deviation $s_{\hat{\gamma}_1}$, $s_{\hat{\gamma}_4}$, and $s_{\hat{\gamma}_5}$, respectively. We randomly draw 10,000 sets of values, including values for variables (Asset Value) and (Future Growth). We then enter each set of these random values into Equation 2 to calculate one conditional coefficient $\hat{\gamma}_{i(1)}$ for each set of values. In this way, we estimate 10,000 possible conditional coefficient values. Only 37 percent of these conditional coefficients are significant; these significant values, however, result in both positive and negative elasticities. The 10th, 50th, and 90th percentiles of these estimated elasticities are $-1.454$, $0.000$, and $+1.432$, respectively.

Figure 1 shows this movement from negative to positive for deciles 10th to 90th. The dotted line is for visual reference. It represents a trend line we created by fitting a regression line to all of the significant results calculated. The progression of (OCF Risk) conditional elasticity from larger negative to larger positive, with a zero value at the 50th percentile is
consistent with the conclusions we draw from Table 2 results, and match the expected results we summarize in Table 1. When neither (Asset Value), nor (Future Growth) dominates, such as at their average sample values, a one percent change in (OCF Risk) results in a 0.000 percent change in (Beta). When (Asset Value) dominates a one percent decrease in (OCF Risk) results in an increase in (Beta) of up to 1.4 percent. Conversely, when (Future Growth) dominates a one percent decrease in (OCF Risk) results in a decrease in (Beta) of up to 1.5 percent.

**Further Investigation of the Estimated Sign of (OCF Risk)**

We investigate further whether asset value and future growth opportunities exert opposing forces on the relationship between operating cash flow risk and beta. We identify a sub-sample that contains firms with concurrently large values for variable (Asset Value) and small values for variable (Future Growth). This sub-sample includes 86 observations. The median for (Asset Value) is $15,800 million and the median for (Future Growth) is 1.21. For this first sub-sample, we expect to find a positive estimated coefficient for (OCF Risk).

Conversely, we identify a second sub-sample that contains firms with concurrently small values for variable (Asset Value) and large values for variable (Future Growth). This sub-sample consists of 107 observations. The median for (Asset Value) is $1,281 million and the median for (Future Growth) is 2.69. For this second sub-sample, we expect to find a negative estimated coefficient for (OCF Risk).

We estimate the following regression model for each of the two sub-samples:

\[
(betta)_{it} = \gamma_1 (OCF Risk)_{it} + \gamma_2 (CFI Risk)_{it} + \gamma_3 (CFF Risk)_{it} + \\
\gamma_4 (Asset Value)_{it} + \gamma_5 (Future Growth)_{it} + \gamma_6 (Bh Debt)_{it} + \\
\gamma_7 (Ph Debt)_{it} + \text{error.} \tag{4}
\]

The estimated results relating to (OCF Risk) are similar under both the fixed and random effects estimators shown in Tables 4 and 5. These results are also consistent with all of our prior results and expectations. The estimated coefficient of (OCF Risk) is positive for the first sub-sample, but negative for the second sub-sample.

The Hausman (1978) test favors the fixed effects results of Table 4 over the random effects results of Table 5. Nevertheless the results of Table 5 support conclusions identical to those of Table 4. In Table 4, for the first sub-sample of large values for (Asset Value) and small values for (Future Growth), the p-value for the estimated coefficient of (OCF Risk) suggests
### Table 4. Fixed Effects Results of the Sub-Sample Observations for Regression Equation (4)\(^a\)

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Large (Asset Value) and Small (Future Growth)</th>
<th></th>
<th>Small (Asset Value) and Large (Future Growth)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated coefficient</td>
<td>p-value</td>
<td>Estimated coefficient</td>
<td>p-value</td>
</tr>
<tr>
<td>OCF Risk</td>
<td>$2.665 \times 10^{-4}$</td>
<td>0.076</td>
<td>$-4.119 \times 10^{-3}$</td>
<td>0.019</td>
</tr>
<tr>
<td>CFI Risk</td>
<td>$-1.703 \times 10^{-4}$</td>
<td>0.079</td>
<td>$3.066 \times 10^{-3}$</td>
<td>0.059</td>
</tr>
<tr>
<td>CFF Risk</td>
<td>$2.867 \times 10^{-5}$</td>
<td>0.421</td>
<td>$-2.586 \times 10^{-3}$</td>
<td>0.103</td>
</tr>
<tr>
<td>Asset Value</td>
<td>$-1.744 \times 10^{-5}$</td>
<td>0.315</td>
<td>$2.871 \times 10^{-4}$</td>
<td>0.000</td>
</tr>
<tr>
<td>Future Growth</td>
<td>$-4.075 \times 10^{-2}$</td>
<td>0.300</td>
<td>$2.735 \times 10^{-2}$</td>
<td>0.134</td>
</tr>
<tr>
<td>Bh Debt</td>
<td>$4.838 \times 10^{-1}$</td>
<td>0.055</td>
<td>$5.537 \times 10^{-2}$</td>
<td>0.416</td>
</tr>
<tr>
<td>Ph Debt</td>
<td>$3.430 \times 10^{-2}$</td>
<td>0.032</td>
<td>$1.601 \times 10^{-1}$</td>
<td>0.090</td>
</tr>
</tbody>
</table>

\(^a\)The dependent variable is the beta of a firm’s shares

### Table 5. Random Effects Results of Sub-sample Observations for Regression Equation (4)\(^a\)

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Large (Asset Value) and Small (Future Growth)</th>
<th></th>
<th>Small (Asset Value) and Large (Future Growth)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated coefficient</td>
<td>p-value</td>
<td>Estimated coefficient</td>
<td>p-value</td>
</tr>
<tr>
<td>OCF Risk</td>
<td>$8.585 \times 10^{-5}$</td>
<td>0.067</td>
<td>$-3.327 \times 10^{-3}$</td>
<td>0.058</td>
</tr>
<tr>
<td>CFI Risk</td>
<td>$-1.759 \times 10^{-5}$</td>
<td>0.449</td>
<td>$2.727 \times 10^{-3}$</td>
<td>0.142</td>
</tr>
<tr>
<td>CFF Risk</td>
<td>$-3.269 \times 10^{-5}$</td>
<td>0.216</td>
<td>$-2.535 \times 10^{-2}$</td>
<td>0.134</td>
</tr>
<tr>
<td>Asset Value</td>
<td>$8.644 \times 10^{-6}$</td>
<td>0.004</td>
<td>$2.953 \times 10^{-6}$</td>
<td>0.000</td>
</tr>
<tr>
<td>Future Growth</td>
<td>$2.245 \times 10^{-1}$</td>
<td>0.000</td>
<td>$3.753 \times 10^{-2}$</td>
<td>0.028</td>
</tr>
<tr>
<td>Bh Debt</td>
<td>$9.530 \times 10^{-1}$</td>
<td>0.000</td>
<td>$3.099 \times 10^{-1}$</td>
<td>0.138</td>
</tr>
<tr>
<td>Ph Debt</td>
<td>$1.071 \times 10^{-2}$</td>
<td>0.344</td>
<td>$1.320 \times 10^{-1}$</td>
<td>0.023</td>
</tr>
</tbody>
</table>

\(^a\)The dependent variable is the beta of a firm’s shares
a 92.4 percent probability of a positive relationship between (OCF Risk) and (Beta). The estimated coefficient for (OCF Risk) is $+2.665 \times 10^{-4}$, which results in an estimated elasticity of +0.135. A one percent decrease in (OCF Risk) results in 0.135 percent decrease in (Beta). On average, this elasticity translates to a 0.06 percent decrease in equity’s cost of capital for every percent decrease in the total operating cash flow risk. We use the CAPM to determine this figure. We use an average risk-free rate of 4 percent and an average market risk-premium of 6 percent. The average beta for this sub-sample is 0.783.

We obtain an opposite result for the second sub-sample of small values for (Asset Value) and large values for (Future Growth). For the second sub-sample in Table 4, the p-value for the estimated coefficient of (OCF Risk) suggests a 98.1 percent probability of a negative relationship between (OCF Risk) and (Beta). The estimated coefficient for (OCF Risk) is $-4.119 \times 10^{-3}$, which results in an estimated elasticity of −0.229. A one percent decrease in (OCF Risk) results in 0.229 percent increase in (Beta). For this sub-sample, this elasticity translates to an average 0.09 percent increase in equity’s cost of capital for every one percent decrease in the total operating cash flow risk. We again use the CAPM, an average risk-free rate of 4 percent, and an average market risk-premium of 6 percent to arrive at this figure. The average beta for this sub-sample is 0.647.

**CONCLUSION**

Risk management textbooks suggest that reductions in total risk sustained by a cash flow volatility reduction program such as reinsurance or hedging do not change a stockholder’s cost of capital. According to this suggestion, there should exist no relationship between a risk management program and a firm’s stock beta. Our empirical results confirm that, for the average insurance firm, this is true. That being said, our empirical results also uncover conditions where a reduction in the total operating cash flow risk leads to either a reduction or an increase in the stock’s beta. Our results suggest that an insurance firm with an asset value that is large in relation to its future opportunities will enjoy a lower cost of equity capital (because of a lower stock beta) when the total risk of operating cash flow is reduced. The reverse seems true for insurance firms where asset value is small in relation to the value of future opportunities. The statistical significance of the estimated relationship between total risk reduction and beta is strong. However, when this estimated relationship is used to calculate changes in a stockholder’s cost of capital using average market values, the economic significance of risk reduction for the average insurer is somewhat weaker.
Of course, in periods where economic conditions are such that the risk-free rate and the market risk premium are high, the economic significance of risk reduction will be pronounced.

While there are firms in the insurance industry with both large and small growth opportunities, stockholders might be searching for a stock with a particular risk profile in order to manage other non-tradable background risks they face. In fact, the model of Franke, Stapleton, and Subrahmanyam (1998) explains the demand for option contracts by risk-averse investors as an attempt to manage their background risk. It could be that there is a clientele for firms with low/high systematic risk. Of course, if managers change the systematic risk of a firm, the firm’s stock will find its place in the portfolios of other investors. But, because investors face information costs, the trading that takes place during a change in clientele increases the stock’s price volatility.

Our results suggest that cash flow volatility reduction programs such as reinsurance or hedging should be individually crafted for each firm and for specific periods in the firm’s life. For example, according to our results, a newly incorporated insurance firm should manage its total risk somewhat less intently, especially during periods where the market risk premium is high. The same firm should manage its total risk more intensely as it matures, especially during periods where the market risk premium is low.

NOTES

1 Risk management is the process of understanding the downside possibilities as well as the upside potential of each of the risks facing the firm, and how these risks as a portfolio affect the firm’s value. One way to mitigate a risk is for the firm’s shareholders to pay some other shareholders in the insurance, capital, or derivatives market to fund the consequences of their firm’s risk.

2 The use of market-based valuation and relative valuation are other techniques for valuing a stock.

3 Consider, for example, a hedge fund. Each fund’s investment strategy is based on the ability of its manager to understand and assess risks better than other investors. Skilled managers will borrow funds so that they can invest in the risks they understand well, meanwhile hedging away risks they feel they have no particular comparative advantage.

4 One potential shortcoming of using the market-to-book ratio as a representation of the firm’s future growth opportunities is that accounting practices can distort the firm’s book value. According to Beaver and Ryan (2000), accounting conservatism and slow recognition of economic gains induce biases and lags in the book value, respectively. Biases increase book value while lags reduce it. Beaver and Ryan (2000) suggest that one interpretation of their results is that over time, firms are subject to a similar extent to both bias and lags. Also, Bauman (1999) concludes that the age of fixed assets is among the most significant contributors to book value distortion. The assets (and liabilities) of the firms in our sample are mainly financial.

5 Reserves represent financial obligations the insurance firm owes to its policyholders. Often, an insurance firm under a separate agreement promises another insurer reimbursement for
some of these obligations. Gross reserves represent the financial obligations an insurance firm owes to its policyholders plus any obligations it owes to other insurers under such separate agreements.

REFERENCES


APPENDIX A—COMPARATIVE STATICS

The change in price of a growth option \((dO)\) embedded in a firm’s assets \((A)\) is:

\[
dO = \frac{\partial O}{\partial A} dA + \frac{\partial O}{\partial t} dt + \frac{1}{2} \frac{\partial^2 O}{\partial A^2} \sigma^2_A dA dt
\]  

(A1)

where \((\sigma^2_A)\) is the total risk of the firm’s assets and \((t)\) is time. Dividing equation (A1) by \((O)\) and taking the limit as \((dt)\) approaches zero, we are left with:

\[
\lim_{dt \to 0} \frac{dO}{O} = \frac{\partial O}{\partial A} \frac{dA}{A O}.
\]  

(A2)

Multiplying and dividing equation (A2) by \((A)\) and rearranging terms, we have:

\[
\frac{dO}{O} = \frac{\partial O}{\partial A} \frac{dA A}{A O}.
\]  

(A3)

Since \((dO/O)\) is the rate of return generated by the growth option \((k_O)\) and \((dA/A)\) is the rate of return generated by the assets of the firm \((k_A)\) we re-write equation (A3) as:

\[
k_O = \frac{\partial O}{\partial A} \frac{A}{O} k_A
\]  

(A4)

The Option Pricing Model (OPM) exists in continuous time, but Hsia (1981) shows the equivalency between the one-period CAPM and Merton’s (1973) continuous time version of the CAPM. Thus, we can relate the one-period CAPM to the OPM. Relying on the one-period CAPM, the beta of the cash flows associated with the option \((\beta_O)\) and the beta of the cash flows associated with the firm’s current assets \((\beta_A)\) are:
where \( \text{Cov}(.) \) is the covariance operator. Substituting equation (A4) into equation (A4a) and recognizing equation (A4b) we have:

\[
\beta_O = \frac{\text{Cov}(k_O;k_m)}{\sigma_m^2} \quad \text{(A4a)}
\]

\[
\beta_A = \frac{\text{Cov}(k_A;k_m)}{\sigma_m^2} \quad \text{(A4b)}
\]

\[
\beta_O = \frac{\text{Cov}\left(\frac{\partial O_A}{\partial A}k_A;k_m\right)}{\sigma_m^2} = \frac{\partial O_A}{\partial A} \frac{\text{Cov}(k_A;k_m)}{\sigma_m^2} = \frac{\partial O_A}{\partial A} \beta_A \quad \text{(A5)}
\]

\[
\beta_O = N(d_1) \frac{A}{O} \beta_A
\]

where, Black and Scholes (1973) explain,

\[
O = AN(d_1) - e^{-k_f T} XN(d_2), \quad \text{(A5a)}
\]

\(N(.)\) is the standardized cumulative normal density function, \(ln(.)\) is the natural logarithm operator, \((X)\) is the exercise price of the option, \((T)\) is the time available to act on the option, and \((k_f)\) is the rate of return on the risk-free asset,

\[
d_1 = \frac{\ln\left(\frac{A}{X}\right) + k_f T + \frac{1}{2} \sigma_A^2 T}{\sigma_A \sqrt{T}} \quad \text{(A5b)}
\]

and

\[
d_2 = d_1 - \sigma_A \sqrt{T}. \quad \text{(A5c)}
\]

From equation (A5) and holding \((\beta_A)\) constant:
\[
\frac{\delta \beta_O}{\delta \sigma_A^2} = A \beta_A \left[ \frac{\delta N(d_1)}{\delta \sigma_A^2} \cdot O - N(d_1) \cdot \frac{\delta O}{\delta \sigma_A^2} \right]. \quad (A6)
\]

Since

\[
\frac{\delta d_1}{\delta \sigma_A^2} = \frac{\ln \left( \frac{A}{X} \right) + k_f T}{\sqrt{T}} \left( -\frac{1}{2 \sigma_A^2} \right) + \frac{1}{4 \sigma_A} = \frac{1}{2 \sigma_A^2} \frac{\ln \left( \frac{A}{X} \right) + k_f T}{\sqrt{T}} \left( -\frac{1}{2 \sqrt{T} \sigma_A} \right) = \frac{1}{2 \sigma_A^2} d_1,
\]

and similarly, \( \frac{\delta d_2}{\delta \sigma_A^2} = \frac{1}{2 \sigma_A^2} d_1 \), based on equation (A5a) the partial derivative of \( O \) is:

\[
\frac{\delta O}{\delta \sigma_A^2} = AZ(d_1) \left( -\frac{1}{2 \sigma_A^2} d_2 \right) e^{-k_f T} X \cdot Z(d_2) \left( -\frac{1}{2 \sigma_A^2} d_1 \right) = \frac{1}{2 \sigma_A^2} \left[ Ad_2 Z(d_1) - e^{-k_f T} X \cdot d_2 \cdot Z(d_2) \right].
\]

The terms in the bracket of equation (A6), after we substitute \( \frac{\delta O}{\delta \sigma_A^2} \) from above, equals:

\[
\frac{1}{2 \sigma_A^2} \left\{ d_2 Z(d_1) \left( AN(d_1) - e^{-k_f T} X \cdot N(d_2) \right) - N(d_1) \left( Ad_2 Z(d_1) - e^{-k_f T} X \cdot d_2 \cdot Z(d_2) \right) \right\} = \frac{1}{2 \sigma_A^2} e^{-k_f T} X \cdot d_1 Z(d_2) N(d_1) - d_2 Z(d_1) N(d_2) =
\]

\[
\frac{1}{2 \sigma_A^2} e^{-k_f T} X \cdot N(d_1) N(d_2) \left( \frac{Z(d_2)}{N(d_2)} - \frac{Z(d_1)}{N(d_1)} \right).
\]

Therefore
\[
\frac{\delta \beta_O}{\delta \sigma_A^2} = \frac{A \beta_A}{O^2} \left( -\frac{1}{2 \sigma_A^2} \right) e^{-k_f T} X \cdot N(d_1) N(d_2) \left( d_1 N(d_2) - d_2 N(d_1) \right); \]

that is:

\[
\frac{\delta \beta_O}{\delta \sigma_A^2} = \frac{Q \sqrt{T}}{2 \sigma_A} \left( \frac{Z(d_2) - Z(d_1)}{d_1 N(d_2) - d_2 N(d_1)} \right) \beta_A, \text{ where} \]

\[
Q = \frac{A e^{-k_f T} X \cdot N(d_1) N(d_2)}{O^2 \sigma_A \sqrt{T}} > 0. \tag{A7}
\]

It can be shown by paralleling the deviation in Galai and Masulis (1976) that \( \frac{Z(d_2)}{d_1 N(d_2) - d_2 N(d_1)} > 0 \) under the mild condition that the firm’s assets are worth at least as much as the market value of its debt, or equivalently, \( A \geq X \cdot e^{-(k_f + \frac{1}{2} \sigma_A^2) T} \). It immediately follows then that \( \frac{\delta \beta_O}{\delta \sigma_A^2} < 0 \) when \( (\beta_A) \) is positive and constant.

To evaluate how \( (\beta_A) \) responds to changes in \( (\sigma_A^2) \) we again use equation (A5) and hold \( (\beta_O) \) constant.

From equation (A5), \( \beta_A = \frac{\beta_O \cdot O}{A N(d_1)} \) and

\[
\frac{\delta \beta_A}{\delta \sigma_A^2} = \frac{\beta_O}{A N(d_1)^2} \left[ \frac{\delta O}{\delta \sigma_A} \frac{N(d_1)}{O} - \frac{\delta N(d_1)}{\delta \sigma_A} \right] =
\]

\[
(-1) \frac{\beta_O}{A N(d_1)^2} \left[ \frac{\delta N(d_1)}{\delta \sigma_A} \cdot O - N(d_1) \cdot \frac{\delta O}{\delta \sigma_A} \right],
\]

it follows that \( \frac{\delta \beta_A}{\delta \sigma_A^2} > 0 \) when \( (\beta_O) \) is positive and constant.
# APPENDIX B—DATA YEARS

<table>
<thead>
<tr>
<th>Firm name</th>
<th>Period</th>
<th>Years of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 21st Century Holding Company</td>
<td>1999 to 2004</td>
<td>6</td>
</tr>
<tr>
<td>2. 21st Century Insurance Group</td>
<td>1993 to 2004</td>
<td>12</td>
</tr>
<tr>
<td>3. ACE Ltd.</td>
<td>2000 to 2004</td>
<td>5</td>
</tr>
<tr>
<td>4. ALFA Corp</td>
<td>1994 to 2004</td>
<td>11</td>
</tr>
<tr>
<td>5. Allmerica Financial Corporation</td>
<td>1996 to 2004</td>
<td>9</td>
</tr>
<tr>
<td>6. Allstate Corporation</td>
<td>1994 to 2004</td>
<td>11</td>
</tr>
<tr>
<td>10. Argonaut Group Inc.</td>
<td>1992 to 2004</td>
<td>13</td>
</tr>
<tr>
<td>11. Atlantic American Corporation</td>
<td>1994 to 2004</td>
<td>11</td>
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<tr>
<td>15. CNA Financial Corporation</td>
<td>1991 to 2004</td>
<td>14</td>
</tr>
<tr>
<td>17. EMC Insurance Group Inc.</td>
<td>1994 to 2004</td>
<td>11</td>
</tr>
<tr>
<td>18. Erie Indemnity Company</td>
<td>1996 to 2004</td>
<td>9</td>
</tr>
<tr>
<td>19. Everest Re Group Ltd.</td>
<td>1996 to 2004</td>
<td>9</td>
</tr>
<tr>
<td>21. Hallmark Financial Services Inc.</td>
<td>1994 to 2004</td>
<td>11</td>
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<tr>
<td>22. Harleysville Group Inc.</td>
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<tr>
<td>23. HCC Insurance Group Inc.</td>
<td>1994 to 2004</td>
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<tr>
<td>24. IPC Holdings Ltd.</td>
<td>1996 to 2004</td>
<td>9</td>
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<tr>
<td>25. Markel Corporation</td>
<td>1993 to 2004</td>
<td>12</td>
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<tr>
<td>26. Merchants Group Inc.</td>
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<td>12</td>
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<tr>
<td>27. Mercury General Corporation</td>
<td>1993 to 2004</td>
<td>12</td>
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<tr>
<td>28. Midland Company</td>
<td>1993 to 2004</td>
<td>12</td>
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<tr>
<td>29. Navigators Group Inc.</td>
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<td>30. NCRIC Group Inc.</td>
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<td>31. NYMAGIC</td>
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<td>32. Ohio Casualty Corporation</td>
<td>1993 to 2004</td>
<td>12</td>
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<td>33. Old Republic International Corporation</td>
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<tr>
<td>34. Partner Re Ltd.</td>
<td>2000 to 2004</td>
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<tr>
<td>35. Penn-America Group Inc.</td>
<td>1993 to 2003</td>
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<td>37. Progressive Corporation</td>
<td>1991 to 2004</td>
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</tr>
<tr>
<td>38. PX Re Corporation</td>
<td>1997 to 2004</td>
<td>8</td>
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<tr>
<td>39. Reinsurance Group of America</td>
<td>1995 to 2004</td>
<td>10</td>
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<td>40. RLI Corporation</td>
<td>1993 to 2004</td>
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<td>41. SAFECO</td>
<td>1991 to 2004</td>
<td>14</td>
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<tr>
<td>42. SCPIE Holdings Inc.</td>
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<td>43. Selective Insurance Group Inc.</td>
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<td>Period</td>
<td>Years of data</td>
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<td>46. St. Paul Travelers Companies Inc.</td>
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<td>47. State Auto Financial Corporation</td>
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<td>48. Transatlantic Holdings Inc.</td>
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<td>49. Unico American Corporation</td>
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<td>50. United Fire and Casualty Company</td>
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<td>51. Unitrin Inc.</td>
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<td>52. Vesta Insurance Group</td>
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<td>53. XL Capital Ltd.</td>
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<td>2000 to 2004</td>
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