How Does the Liability Structure Affect Incentives to Invest in Hedge Funds? The Case of With-Profit Life Insurance

Thomas R. Berry-Stölzle, Hendrik Kläver, and Shen Qiu

Abstract: This paper investigates the impact of a life insurer’s liability structure on its incentives to invest in hedge funds. The analysis is based on a simulation model of a life insurance company selling with-profit life insurance policies with cliquet-style interest rate guarantees. The insurer has three investment alternatives: stocks, bonds, and a hedge fund. We maximize the insurer’s investment portfolio returns for given levels of insolvency risk measured by the insurer’s shortfall probability. A profit-maximizing insurer has incentives to invest in hedge funds if such investments increase the portfolio return for a given shortfall probability. The focus of our analysis is on incentive differences for different liability parameterizations. Our analysis reveals, first, that insurers with high interest rate guarantees, and financially weak insurers, have a higher incentive to invest in hedge funds, and, second, the optimal hedge fund holding of these insurers is very sensitive to expectations about future hedge fund returns. This interdependence is of importance to insurance companies, stakeholders, and especially regulators, since the strong incentives for financially weak insurers might create a moral hazard to overinvest. [Key words: Life insurance, hedge funds, over-investment.]
INTRODUCTION

Life insurance policies with a minimum interest rate guarantee and a surplus participation mechanism—so-called with-profit life insurance policies—have a significant market share in many countries, including the United States, Japan, Great Britain, and Germany. The guarantees and the resulting obligations for the insurance companies inherent in these policies vary. Some policies include point-to-point guarantees for the surplus participation, where the insurer credits a certain amount to the policyholder’s account every year, but the accumulated surplus participation is only guaranteed at maturity of the contract (see, e.g., Briys and de Varenne, 1997). This mechanism is also referred to as conditional bonus (Grosen and Jorgensen, 2002). For an insurer offering point-to-point guarantees, it is, hence, tolerable if its investment portfolio is not able to cover the complete policyholder’s account in a given year, as long as all obligations are met in the end. Other policies include a cliquet-style (or year-by-year) guarantee that locks in any return credited to the policyholder’s account on a yearly basis. Each year, the guaranteed interest rate has to be applied to the total policyholder’s account balance. Such a guarantee restricts the possible asset management strategies of a life insurance company substantially, since the insurer cannot offset lower than guaranteed investment returns in one year by higher returns in later years, but has to meet its obligations every year.

This paper focuses on with-profit life insurance policies including a cliquet-style interest rate guarantee. We are especially interested in how the insurance company’s liability structure affects its decision to invest in hedge funds. Since the early 1990s, hedge funds have become increasingly popular with both institutional investors and high-net-worth individuals.\(^5\) Like mutual funds, hedge funds provide actively managed portfolios in publicly traded assets. The main difference between hedge funds and mutual funds is that hedge funds are less regulated (Brown and Goetzmann, 2003; Fung and Hsieh, 1999).\(^6\) Therefore hedge funds can freely design incentive contracts with investment managers and endow them

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\(^5\)Assets under management in the hedge fund industry rose from approximately $50 billion in 1990 to approximately $1 trillion by the end of 2004 (Malkiel and Saha, 2005). Assets under management peaked at about $2.5 trillion at the beginning of 2008, just before the financial crisis hit the industry (Ineichen and Silberstein, 2008).Withdrawals of assets combined with market losses reduced assets under management to about $1.56 trillion in November 2008, according to data compiled by Hedge Fund Research Inc. in Chicago. More recently, Hedgefund.net reports that in August 2010 hedge funds have slightly over $2.2 trillion in assets under management.
with substantial discretion by restricting investors’ rights to withdraw their money (Agarwal, Daniel, and Naik, 2009). Furthermore, hedge funds can freely choose the type of securities in which they invest as well as the type of positions they take. For example, hedge funds can invest in derivative securities, sell short, and take on leveraged or undiversified positions (see, e.g., Fung and Hsieh, 1997a; Liang, 2000). Such non-standard investment strategies have the potential to generate risk return profiles different from traditional asset classes, making them an interesting investment alternative for institutional investors like life insurance companies.

Research to date in hedge fund investing has primarily focused on evaluating hedge fund risk (e.g., Ackermann, McEnally, and Ravenscraft, 1999; Brown, Goetzmann, and Ibbotson, 1999; Edwards and Caglayan, 2001; Liew, 2003; Agarwal and Naik, 2004), constructing optimal portfolios of hedge funds (e.g., McFall, 2003; Kat, 2004; Agarwal and Naik, 2004; Morton, Popova, and Popova, 2006; Giamouridis and Vrontos, 2007), and determining the optimal hedge fund holding in a portfolio consisting of different asset classes (e.g., Amin and Kat, 2002; Cvitanic, Lazrak, Martellini, and Zapatero, 2003; Popova, Morton, and Popova, 2003; Terhaar, Staub, and Singer, 2003; Boyle and Liew, 2007). We extend this latter class of investment models by developing a full-fledged simulation model of a life insurance company incorporating both its investments and its liabilities resulting from the life insurance business.

Previous portfolio models focusing on hedge fund investments have applied various approaches, ranging from the standard mean-variance analysis (e.g., Amenc and Martellini, 2002; Terhaar, Staub, and Singer, 2003; Alexander and Dimitriu, 2004) and the microeconomic expected utility framework (e.g., Barès, Gibson, and Gyger, 2002; Cvitanic, Lazrak, Martellini, and Zapatero, 2003) to higher moment analysis (e.g., Popova, Morton, and Popova, 2003; Davies, Kat, and Lu, 2005), and portfolio selections using alternatives to the standard variance-risk measures such as the conditional value at risk and generalized measures (e.g., Krokhmal, Uryasev, and Zrazhevsky, 2002; Agarwal and Naik, 2004; Rockafellar, Uryasev, and

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6In the United States, for example, hedge funds are free from the regulatory controls stipulated by the Investment Company Act of 1940.

7Compensation contracts of hedge fund managers typically include incentive fees with a hurdle rate and high-water-mark provisions (see, e.g.: Agarwal, Daniel, and Naik, 2009). With a hurdle rate provision, a manager earns an incentive fee only if the fund returns exceed the hurdle rate; with a high-water-mark provision, a manager earns an incentive fee only on new profits, i.e., after recovering past losses. Share restrictions that limit the liquidity of fund investors include lockup periods and redemption notice periods. During the lockup period an investor cannot withdraw the money. After the lockup period, an investor who wants to withdraw has to give advance notice.
In the same vein, we construct investment portfolios for our model life insurance company using the company’s shortfall probability as a risk measure.8

In recent years, a series of studies has addressed different types of interest rate guarantees in with-profit life insurance policies, specifically, point-to-point guarantees (e.g., Briys and de Varenne, 1997; Grosen and Jorgensen, 2002), cliquet-style guarantees (e.g., Grosen and Jorgensen, 2000; Jensen, Jorgensen and Grosen, 2001; Miltersen and Persson, 2003; Hansen and Miltersen, 2002) and guarantees inherent in the most common policy designs of a specific country, such as the UK (Ballotta, Haberman, and Wang, 2006) or Germany (Bauer, Kiesel, Kling, and Ruß, 2006). The primary focus of this literature is on the risk-neutral valuation of life insurance contracts. However, Kling, Richter, and Ruß (2007a, 2007b) examine the effect a policy has on the risk exposure of an insurance company, measured by the shortfall probability under the so-called “real-world probability measure P.” Assuming a cliquet-style interest rate guarantee, they study how an insurance company’s ruin probability depends on its policy design, its surplus distribution mechanism, its reserve situation, and its management decisions, as well as on regulatory constraints.

In assessing the impact of a life insurer’s liability structure on its incentives to invest in hedge funds, we build on the model framework developed by Kling, Richter, and Ruß (2007a). We extend this model on the asset side by incorporating three correlated AR(1) GARCH(1,1) processes and an optimization procedure to determine firm-risk investment-return efficient investment portfolios. We calibrate the model to empirically reasonable parameter values and examine the difference between efficient investment portfolios with and without hedge funds in reference to the insurance company’s shortfall probability under various parameter settings reflecting the company’s product characteristics as well as its operating environment.

To clarify, the main focus of our analysis is neither on the development of an econometric hedge fund return model per se nor on the development of a detailed liability model of a life insurance company, including various types of guarantees. The main focus of our analysis is on the interaction effects between a life insurer’s liability structure and its incentives to investment in hedge funds. We are especially interested in the comparative

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8We use the shortfall probability rather than the expected shortfall because the focus of our analysis is on insurance companies’ economic incentives to invest in hedge funds. In this context, the severity of a shortfall should not matter. If the insurance company goes bankrupt, owners’ shares are worthless and managers lose their jobs regardless of how big the remaining deficit of the company is.
statics of life insurer’s liability structures. How does an insurer’s capitalization affect its incentives to invest in hedge funds? Do insurers with higher or lower interest rate guarantees have stronger incentives to invest in hedge funds? Are these incentive differences sensitive to changes in expected hedge fund return characteristics? Providing some answers to these important questions is what we see as the contribution of our study. To our knowledge, this is the first study analyzing the relationship between a life insurer’s liability structure and its hedge fund holdings.

The remainder of this paper is organized as follows. First, we present our model and describe how we calibrate it. We then present our empirical results and discuss their implications for the asset management of insurance companies as well as for insurance regulators. The final section concludes.

MODEL

In this section, we introduce our model, which is a selective simplification of a life insurance company offering a product with a cliquet-style interest rate guarantee. Our model builds on the basic Kling, Richter, and Ruß (2007a) framework. We extend this framework incorporating a more realistic model for the investment portfolio of the insurance company, keeping the original model for the insurance company’s liabilities as is.9 While specific features of the model are motivated by the German regulatory environment,10 these could easily be modified to reflect the regulatory situation in other countries.

9Since empirical studies of hedge fund returns found positive autocorrelation and a larger kurtosis than that of the lognormal distribution (Agarwal and Naik, 2004; Brooks and Kat, 2002; Fung and Hsieh, 1999a, 2001; Martin, 2001; Popova, Morton, and Popova, 2003), the geometric Brownian motion employed by Kling, Richter, and Ruß (2007a) is not appropriate to describe hedge fund returns. Our AR(1) GARCH(1,1) asset model explicitly accounts for return autocorrelations and excess kurtosis. After calibrating our model to the German stock index (DAX), the German bond index (REXP), and the Hedge Funds Indices of the Center for International Securities and Derivatives Markets (CISDM), the null hypothesis that observed and simulated returns have the same distributions cannot be rejected by means of a Kolmogorov-Smirnov test and standard significance levels.

10In 2004, the German Insurance Authority (BaFin) lifted the ban on hedge fund investments for German life insurance companies. Life insurers are now allowed to invest up to five per cent of their assets in hedge funds, using the opening clause up to ten percent. In addition, insurers can file an application with the Insurance Authority to increase the maximum hedge fund holding by another five percent to a total of 15 percent. Thus, for German life insurance companies, it is a timely question whether they should invest in hedge funds or not.
We use a simplified balance sheet perspective as visualized in Figure 1. $A_t$ denotes the market value of the insurance company’s assets, and $L_t$ denotes the book value of the insurer’s technical reserves. $L_t$ results from the insurer’s obligations to its policyholders. Therefore, we will also refer to $L_t$ as the book value of the policyholders’ account. We further assume that $L_t$ corresponds to the book value of the insurance company’s assets. Thus, the reserve account $R_t$, which is given by $R_t = A_t - L_t$, reflects the difference between the market value and the book value of the assets. $R_t$ may also include the insurer’s capital and other additional components; however, we will refer to $R_t$ as the hidden reserves or “the reserves.”

**The Insurance Company’s Assets**

Our model insurance company is invested in a mix of three different assets: a stock, a bond, and a hedge fund. Let $q_1$, $q_2$, and $q_3$ denote the fraction of assets the insurance company invests in the three different investment instruments.\(^1\) We assume that regulatory requirements do not allow short-sales or borrowing for investment purposes, so $0 \leq q_i \leq 1$, $i = 1, 2, 3$ and $\Sigma_{i=1}^{3} q_i = 1$. These assumptions are cemented in the legislation of many countries (see, e.g., the German regulatory law). Let $S_{1,t}$, $S_{2,t}$ and $S_{3,t}$, $t = 1, \ldots, T$ denote the price processes of the stock, bond, and hedge fund, respectively. If we set the prices of the three assets equal to one at time $t = 1$ the value of the insurance company’s assets at time $t = 2, \ldots, T$ is determined by

$$A_t = A_1(q_1 S_{1,t} + q_2 S_{2,t} + q_3 S_{3,t}).$$

\(^1\)The model assumes no rebalancing between asset classes.
The development of the three asset price processes \( S_{1,t}, S_{2,t}, \) and \( S_{3,t} \) over time is determined by three correlated AR(1) GARCH(1,1) processes of the corresponding standardized continuous rates of returns. Let

\[
\begin{align*}
    r_{i,t} &= \frac{\ln \left( \frac{S_{i,t}}{S_{i,t-1}} \right) - E \left( \ln \left( \frac{S_{i,t}}{S_{i,t-1}} \right) \right)}{\sigma \left( \ln \left( \frac{S_{i,t}}{S_{i,t-1}} \right) \right)} \\
\end{align*}
\]

(2)
denote the standardized continuous rate of return of asset \( i = 1, 2, 3 \). We model \( r_{i,t} \) as an AR(1) process

\[
    r_{i,t} = a_{i,0} + a_{i,1} r_{i,t-1} + Z_{i,t}
\]

(3)
where \( Z_{i,t} \) follows a GARCH(1,1) process with conditional variance

\[
    h_{ii,t} = \alpha_0 + \alpha_1 Z_{i,t-1}^2 + \beta_1 h_{ii,t-1}
\]

(4)
and conditional covariance for \( i \neq j \)

\[
    h_{ij,t} = \rho_{ij} \alpha_0 + \alpha_1 Z_{i,t-1} Z_{j,t-1} + \beta_1 h_{ij,t-1}.
\]

(5)
Then

\[
    H_t = \begin{bmatrix} h_{11,t} & h_{12,t} & h_{13,t} \\ h_{12,t} & h_{22,t} & h_{23,t} \\ h_{13,t} & h_{23,t} & h_{33,t} \end{bmatrix}
\]

(6)
is the conditional covariance matrix, and \( H_t \) is positive definite for all \( t \in \mathbb{N} \) if \( H_0 \) is positive definite. This follows from a more general result of Engle and Kroner (1995).

Our model is easy to simulate. Let \( \varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t})' \) be independent and identically \( N(0, I_3) \)-distributed random vectors for all \( t \in \mathbb{Z} \); here \( I_n \) is the \( n \)-dimensional identity matrix.\(^{12}\) Let \( (Z_{1,t}, Z_{2,t}, Z_{3,t})' = C_t \varepsilon_t \) where \( C_t \) is a positive definite matrix satisfying \( C_t^2 = H_t \), i.e., \( C_t \) is a root of \( H_t \). Then each of the time series \( (Z_{1,t})_{t \in \mathbb{Z}}, (Z_{2,t})_{t \in \mathbb{Z}}, \) and \( (Z_{3,t})_{t \in \mathbb{Z}} \) follows a GARCH(1,1) process.
(Zit)\(t \in \mathbb{Z}, i = 1, 2, 3\), are weakly stationary if and only if \(\alpha_1 + 2\beta_1 < 1\). For every \(t\) and \(i \neq j\) the unconditional correlation between \(Z_{it}\) and \(Z_{jt}\) is \(\rho_{ij}\). This procedure allows us to generate three correlated GARCH(1,1) processes which are then used as input for equation (3). However, the conditional correlations

\[
\frac{1}{\sqrt{h_{ii,t} h_{jj,t}}} \left[ \frac{h_{ij,t}}{h_{ii,t}} \right]
\]

for \(i \neq j\) depend on \(t\). For a more general approach and details concerning multivariate GARCH, see Engle and Kroner (1995).

If the insurance company pays dividends to equity holders at time \(t\) the insurance company’s overall assets are reduced by the dividend payments \(D_t\). Let \(A_t^-\) denote the value of the insurance company’s assets before dividend payments and \(A_t^+\) the value of the company’s assets after dividend payments. Then the company’s assets are recursively given by

\[
A_t^- = \frac{A_{t-1}^+}{(q_1 S_{1,t-1} + q_2 S_{2,t-1} + q_3 S_{3,t-1})} (q_1 S_{1,t} + q_2 S_{2,t} + q_3 S_{3,t})
\]

(7)

and

\[
A_t^+ = A_t^- - D_t.
\]

(8)

The Insurance Company’s Liabilities

The development of an insurance company’s liabilities over time depends on multiple factors: the products the company sells, the legal environment the company operates in, the accounting rules applicable, specific management decisions, and, last but not least, the development of the insurance company’s investment portfolio. To model such a complex process, we need a number of simplifying assumptions. We use the model of Kling, Richter, and Ruß (2007a) to model a life insurance company’s liabilities, and the remainder of this section describes their original assumptions.

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12We use Matlab’s RANDN command to generate normally distributed random numbers. Applying the Kolmogorov-Smirnov test, we were not able to reject the null hypothesis that the generated random numbers are normally distributed.

13It is assumed that dividends reduce the investment in all asset classes proportionally.
framework. The model insurance company is a standard German life insurance company offering one simple product: a single-premium fixed term insurance contract. The premium \( P \) is paid up front at time \( t = 0 \), and the benefit is paid at the end of the fixed contract period \( T \) regardless of whether the insured person is still alive or not. Therefore, there are no mortality effects considered in our model. The benefit of this contract depends only on the development of the policyholder’s account over time and is given by \( P \frac{LT}{L_0} \). Ignoring mortality effects in a model of a life insurance company might seem to be inappropriate. However, if we assume that the insurance company’s new business compensates for mortality and policy surrenders, the liabilities resulting from a fixed term contract in combination with the corresponding assets provide a good approximation for a whole insurance company. Furthermore, abstracting from mortality effects allows us to focus on the impact of the insurer’s investment decisions on its liabilities and, hence, its overall risk exposure.

The following features of the model reflect the legal and regulatory situation in Germany. However, the specifics of other countries’ legal environments could be incorporated analogously.

In Germany, there is a cap on the rate of return that may be used to discount technical reserves.\(^{14}\) Almost all life insurance products sold on the German market use the maximum rate of return to discount the reserves. Since the surrender values of life insurance products are legally required to correspond to the value of the technical reserves, policyholders have a year-by-year guarantee of earning at least this rate of return on their account value.\(^{15}\) Currently, the guaranteed interest rate is 2.25%. However, these cliquet-style interest rate guarantees are given at the beginning for the whole contract period and cannot be changed retroactively.\(^{16}\) Therefore, most contracts in the portfolio of a typical German life insurance company are still entitled to 2.75%, 3.25%, or even 4% interest per year. In our model, here, \( g \) denotes the guaranteed interest rate.

Furthermore, at least 90% of the investment returns on a book value basis have to be credited to the policyholders’ accounts.\(^{17}\) Historically, the guaranteed interest rate was much smaller than the market interest rates,

\(^{14}\)Verordnung über Rechnungsgrundlagen für die Deckungsrückstellung, or DeckRV (provision regarding the actuarial basis of life insurance reserves).

\(^{15}\)§169 Abs. 3 VVG (insurance contract law).

\(^{16}\)§2 Abs. 2 DeckRV.

\(^{17}\)§4 Abs. 3 of the Verordnung über die Mindestbeitragsrückerstattung in der Lebensversicherung, or MindZV (provision regarding the minimum surplus participation in life insurance).
so this legal requirement was introduced to give policyholders a fair share of the surplus earned with their money. In our model \( \delta \) represents such a “minimum participation rate.” The minimum participation is based on book values, and, hence, not every rise in the market value of the assets increases the insurer’s obligations to its policyholders. The management of the insurance company has some discretion whether to use certain accounting rules such as the lower of cost and market principle which avoids showing increases in asset values on the balance sheet. However, there are restrictions to the extent to which a company can redirect investment gains into hidden reserves. In our model at least the fraction \( y \) of rises in market values has to be shown on the balance sheet. Thus, the insurance company has to credit at least the amount \( \delta \cdot y \cdot (A_t - A_{t-1}) \) to the policyholders’ accounts.

Usually German insurers credit a smoothed rate of interest to the policyholders’ accounts. This interest rate is typically much higher than the guaranteed rate. Insurers are motivated to hold their overall interest rate as constant as possible to signal stability and low risk compared to other investment instruments—for example, mutual funds. To model such a management decision rule explicitly, we assume that the insurance company’s management monitors the company’s reserve quota \( x_t = \frac{R_t}{L_t} \), and takes actions to keep \( x_t \) within a pre-specified range \([a, b]\). As long as \( a \leq x_t \leq b \) the insurer credits a target rate of interest \( z > g \) to the policyholder’s account. Thus, the value of the policyholder’s account \( L_t = (1 + z)L_{t-1} = (1 + g)L_{t-1} + (z - g)L_{t-1} \) increases by the guaranteed interest rate and by a fraction of the company’s surplus. We further assume that the insurance company pays a fraction \( d \) of any surplus credited to the policyholders as dividends to its shareholders. If the reserve quota \( x_t \) falls below \( a \), the insurer first reduces the target interest rate \( z \) as well as the dividend payments. If the insurer lacks sufficient investment income and hidden reserves to provide the interest rate guarantees, it becomes insolvent. More precisely, at every point in time \( t = 1, ..., T \) we distinguish between the following five cases:

- **Case 1:** The insurance company does not have enough assets to provide the interest rate guarantees: \( A_t < (1 + g)L_{t-1} \). If this happens for one \( t = 1, ..., T \), the company is bankrupt.

- **Case 2:** The insurance company has enough assets to fulfill its interest rate guarantees but cannot meet the management goal to hold the reserve quota within the range \([a, b]\): \((1 + g)L_{t-1} \leq A_t \leq (1 + a)(1 + g)L_{t-1}\). In this case the insurer credits only the guaranteed interest rate to the
policyholders’ accounts and does not pay dividends to its shareholders: \(L_t = (1 + g)L_{t-1}\) and \(D_t = 0\).

- **Case 3:** If the insurance company credits the target rate of interest \(z\) to the policyholders’ accounts, the reserve quota would fall below \(a\); but if the insurer credits only the guaranteed rate \(g\) to the policyholders’ accounts, the reserve quota would be higher than \(a\): \((1 + a)(1 + g)L_{t-1} < A_t < [(1 + a)(1 + z) + d(z - g)]L_{t-1}\). In this case, the insurance company credits an interest rate \(u \in (g, z)\) to the policyholders’ accounts such that the resulting reserve quota \(x_t = a\). Then the insurer’s liabilities rise to \(L_t = \frac{A_t + d(1 + g)L_{t-1}}{1 + a + d}\), and the insurer pays dividends
\[
D_t = d\frac{A_t - (1 + a)(1 + g)L_{t-1}}{1 + a + d}.
\]

- **Case 4:** If the insurer credits the target rate of interest \(z\) to the policyholders’ accounts, the reserve quota \(x_t\) stays in the pre-specified range \([a, b]\): \([(1 + a)(1 + z) + d(z - g)]L_{t-1} \leq A_t \leq [(1 + b)(1 + z) + d(z - g)]L_{t-1}\). In this case, the insurer credits the target interest rate \(z\) to the policyholders’ accounts and distributes a fraction of its surplus to the shareholders: \(L_t = (1 + z)L_{t-1}\) and \(D_t = d(z - g)L_{t-1}\).

- **Case 5:** The insurance company does extremely well. If the company credits the target interest rate \(z\) to the policyholders’ accounts, there would be more capital in the company than desired by the management \((x_t > b)\): \(A_t > [(1 + b)(1 + z) + d(z - g)]L_{t-1}\). In this case, the insurance company credits a higher interest rate \(v \in (z, \infty)\) to the policyholders’ accounts such that the resulting reserve quota \(x_t = b\). Then the insurer’s liabilities rise to \(L_t = \frac{A_t + d(1 + g)L_{t-1}}{1 + b + d}\), and the insurer pays also higher dividends \(D_t = d\frac{A_t - (1 + b)(1 + g)L_{t-1}}{1 + b + d}\) to shareholders.

To ensure that our model insurance company does not violate the legally required minimum participation of policyholders on book value investment returns, we explicitly check in the cases 2, 3, 4, and 5 whether the company credits at least \(\delta \cdot y \cdot (A_t - A_{t-1})\) to the policyholders’ accounts. If the insurance company violated this condition and \(L_t - L_{t-1} < \delta \cdot y \cdot (A_t - A_{t-1})\), we will enforce the minimum participation by setting \(L_t = L_{t-1} + \delta \cdot y \cdot (A_t - A_{t-1})\) and adjusting the dividend payment \(D_t = d[\delta \cdot y \cdot (A_t - A_{t-1}) - gL_{t-1}]\).
Efficient Asset Allocations

The model described so far determines, among other things, whether the insurance company goes bankrupt. Using this model in a Monte Carlo simulation, we can calculate the shortfall probability of the company. For a given asset allocation and given asset scenarios, we can also calculate the expected rate of return of the insurance company’s investment portfolio. Therefore, we can interpret the whole model as a function

\[
\begin{pmatrix}
\text{shortfall probability} \\
E(\text{portfolio return})
\end{pmatrix} = f(\text{asset allocation}, \text{assetscenarios}, \text{others}) \tag{9}
\]

of the asset allocation, the asset scenarios and the parameters reflecting the product characteristics, and the company’s operating environment. Since this function returns a risk as well as a return measure, we can derive a risk-return efficient frontier, assuming that short sales are not allowed. The insurance company can then choose a portfolio from the efficient set according to its risk preferences.

The focus of our analysis is on the impact of a life insurer’s liability structure on incentives to invest in hedge funds. If the investment returns of an insurer based on efficient portfolios that actually include a positive hedge fund position are greater than the investment returns based on the corresponding efficient portfolios without hedge funds, then this insurer has an incentive to invest in hedge funds. The larger the return difference, the higher the incentive to invest in hedge funds. Thus, we use the difference between the efficient frontiers with and without hedge funds as a proxy for such incentives. We calculate efficient frontiers with and without hedge funds for various parameter sets describing the insurer’s liability structure. This procedure allows us to examine how differences in a life insurer’s liability structure affect its incentives to invest in hedge funds.

DATA AND MODEL CALIBRATION

In our numerical analysis, we calibrate the model to empirically reasonable parameter values. To ensure the stability of our results, we analyze three additional sets of parameters for our asset model reflecting more negative expectations about the future hedge fund returns.
Table 1. Descriptive Statistics and Correlations of Asset Return Time Series

<table>
<thead>
<tr>
<th></th>
<th>Mean (annual.)</th>
<th>Std. dev. (annual.)</th>
<th>Correlation with DAX</th>
<th>Correlation with REXP</th>
<th>Corr. based on returns ≤ median with DAX</th>
<th>Corr. based on returns ≤ median with REXP</th>
<th>Skew</th>
<th>Excess kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convertible arbitrage</td>
<td>0.100</td>
<td>0.027</td>
<td>0.198</td>
<td>0.149</td>
<td>0.112</td>
<td>0.100</td>
<td>-1.304</td>
<td>2.634</td>
</tr>
<tr>
<td>Distressed securities</td>
<td>0.121</td>
<td>0.057</td>
<td>0.463</td>
<td>0.048</td>
<td>0.356</td>
<td>0.206</td>
<td>-1.647</td>
<td>9.545</td>
</tr>
<tr>
<td>Event-driven</td>
<td>0.129</td>
<td>0.050</td>
<td>0.550</td>
<td>-0.022</td>
<td>0.500</td>
<td>0.193</td>
<td>-1.134</td>
<td>5.290</td>
</tr>
<tr>
<td>Equity long/short</td>
<td>0.129</td>
<td>0.077</td>
<td>0.581</td>
<td>-0.033</td>
<td>0.479</td>
<td>0.120</td>
<td>-0.350</td>
<td>3.502</td>
</tr>
<tr>
<td>Emerging markets</td>
<td>0.106</td>
<td>0.133</td>
<td>0.455</td>
<td>-0.136</td>
<td>0.466</td>
<td>0.058</td>
<td>-3.399</td>
<td>25.758</td>
</tr>
<tr>
<td>Equity market neutral</td>
<td>0.087</td>
<td>0.019</td>
<td>0.347</td>
<td>0.095</td>
<td>0.290</td>
<td>0.150</td>
<td>0.031</td>
<td>2.183</td>
</tr>
<tr>
<td>Global macro</td>
<td>0.099</td>
<td>0.061</td>
<td>0.363</td>
<td>0.355</td>
<td>0.183</td>
<td>0.325</td>
<td>0.936</td>
<td>3.364</td>
</tr>
<tr>
<td>Merger arbitrage</td>
<td>0.104</td>
<td>0.037</td>
<td>0.458</td>
<td>0.007</td>
<td>0.464</td>
<td>0.181</td>
<td>-1.229</td>
<td>8.745</td>
</tr>
<tr>
<td>Equal weighted</td>
<td>0.136</td>
<td>0.070</td>
<td>0.607</td>
<td>-0.021</td>
<td>0.553</td>
<td>0.110</td>
<td>-0.501</td>
<td>4.082</td>
</tr>
<tr>
<td>DAX</td>
<td>0.084</td>
<td>0.229</td>
<td>1.000</td>
<td>-0.130</td>
<td></td>
<td></td>
<td>-0.842</td>
<td>2.674</td>
</tr>
<tr>
<td>REXP</td>
<td>0.068</td>
<td>0.032</td>
<td>-0.130</td>
<td>1.000</td>
<td></td>
<td></td>
<td>-0.326</td>
<td>-0.030</td>
</tr>
</tbody>
</table>
We use monthly return data from the German stock index (DAX), the German bond index (REXP), and the CASAM CISDM hedge fund strategy indices from the Center for International Securities and Derivatives Markets (CISDM) for the months 01/1992 through 12/2005 to calibrate our model. Note that our data do not include the 2007–2009 financial crisis. A description of the investment strategies underlying the various CASAM CISDM hedge fund style indices is provided in the Appendix. More precisely, for one set of simulations we use the DAX, the REXP, and one of the hedge funds indices. For each of these three empirical time series we then calculate the mean and standard deviation and standardize the time series by subtracting the mean and dividing by the standard deviation. For each pair of time series \( i \neq j \in \{1, 2, 3\} \) we calculate the correlation coefficient \( \rho_{ij} \). The first four columns of Table 1 present the annualized means, standard deviations, and correlation coefficients of the return time series. We then estimate the AR(1) parameter \( \alpha_{i,1} \) separately for each time series \( i = 1, 2, 3 \) and determine one pair of GARCH(1,1) parameters \( \alpha_1 \) and \( \beta_1 \) for all three time series—because in the described model all time series must have the same parameters. Since the estimated GARCH parameter values for the REXP are very small, we set \( \alpha_1 \) and \( \beta_1 \) to the average of the parameter estimates for the DAX and the hedge funds time series. The GARCH processes are weakly stationary if \( \alpha_1 + \beta_1 < 1 \), which was the case with all our triples consisting of DAX, REXP, and one of the CASAM CISDM hedge fund strategy indices. We calculate the parameter \( \alpha_0 = 1 - \alpha_1 - \beta_1 \) to ensure that the unconditional variance \( \text{Var}(Z_{it}) \) is equal to one. Table 2 summarizes the AR(1) GARCH(1,1) parameters of our asset model.

In the simulation procedure of the time series we first simulate the GARCH series. As starting values, we simulate a 3-dimensional normal distribution with the given correlation coefficients. The further values of the time series are determined by calculating the conditional covariance matrix \( H_t \), deriving the root \( C_t \) and multiplying it with normally distributed innovations \( \varepsilon_t \). Starting from these GARCH processes, we build the AR processes, multiply the standard deviation and add the mean to each observation. This results in a set of simulated series of the DAX, REXP, and hedge funds.

We use the last step of our simulation procedure—multiplying by the standard deviation and adding the mean of the return time series to the simulated processes—to fine-tune the model. Since the estimated mean return for bonds based on the 1992–2005 data is 6.8% per year, which seems to be too high compared to the current low interest rates, we reduce the expected return for bonds to 5%.18 We also address the well-documented biases present in hedge fund databases (e.g., Ackermann, McEnally, and Ravenscraft, 1999; Brown, Goetzmann, and Ibbotson, 1999; Liang, 1999;
Fung and Hsieh, 2000) at this step in the simulation process. The most prominent bias is the survivorship bias, which results from the fact that hedge funds can choose voluntarily whether they report their return data to a database. Thus, poor-performing hedge funds will probably stop reporting their returns.

In their empirical analysis of hedge fund returns, Fung and Hsieh (2000) find a survivorship bias of 3%. Brown, Goetzmann, and Ibbotson (1999) also find a survivorship bias of 3% for offshore hedge funds. Other studies report a survivorship bias of smaller magnitude.\(^\text{19}\) An additional selection bias may exist if only hedge funds with good performance choose to report their performance to the database vendor in the first place. However, Fung and Hsieh (2000) argue that very successful hedge funds

\(^\text{18}\)The Deutsche Bundesbank reports interest rates for various bonds on a monthly basis. For the 01/1992 through 12/2005 period, the average interest rate for bonds with remaining time to maturity between three and four years is 4.78% and the average interest rate for bonds with remaining time to maturity between nine and ten years is 5.50%. Therefore, we set the expected return for bonds to 5% in our model.

\(^\text{19}\)Liang (2000) reports that the survivorship bias for hedge funds is over 2% per year. Baquero, ter Horst, and Verbeek (2005) find a survivorship bias of 2.11% per year. Edwards and Caglayan (2001) report a survivorship bias of 1.85%. Fung and Hsieh (1997b) find a survivorship bias of 1.5%. Ackermann, McEnally, and Ravenscraft (1999) report that the survivorship bias is quite small, averaging 0.013% per month or 0.16% per year.

### Table 2. Parameter Values for Asset Processes

<table>
<thead>
<tr>
<th></th>
<th>AR(1)</th>
<th></th>
<th>GARCH(1,1)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a_{i0})</td>
<td>(a_{i1})</td>
<td>(\alpha_0)</td>
<td>(\alpha_1)</td>
</tr>
<tr>
<td>Convertible arbitrage</td>
<td>0.019</td>
<td>0.612</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>Distressed securities</td>
<td>-0.010</td>
<td>0.334</td>
<td>0.02</td>
<td>0.10</td>
</tr>
<tr>
<td>Event-driven</td>
<td>-0.012</td>
<td>0.367</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>Equity long/short</td>
<td>-0.036</td>
<td>0.243</td>
<td>0.03</td>
<td>0.15</td>
</tr>
<tr>
<td>Emerging markets</td>
<td>0.052</td>
<td>0.286</td>
<td>0.03</td>
<td>0.21</td>
</tr>
<tr>
<td>Equity market neutral</td>
<td>-0.005</td>
<td>0.330</td>
<td>0.05</td>
<td>0.12</td>
</tr>
<tr>
<td>Global macro</td>
<td>-0.056</td>
<td>0.101</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>Merger arbitrage</td>
<td>-0.010</td>
<td>0.362</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Equal weighted</td>
<td>-0.008</td>
<td>0.311</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>DAX</td>
<td>0.047</td>
<td>-0.026</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>REXP</td>
<td>-0.002</td>
<td>0.208</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
</tbody>
</table>
that are already closed to new investors also may decide not to disclose their performance. Therefore, they conclude that the selection bias is negligently small, if it exists at all. An instant history bias may exist because when a vendor adds a new hedge fund to a database, historical returns for this fund are often back-filled.\textsuperscript{20} However, since the indices we use to calibrate our model are based only on hedge fund returns from funds that have reported these returns at the time the index is calculated, the instant history bias is not relevant to our study.\textsuperscript{21} In summary, to reflect the only relevant bias for our study—the survivorship bias—we subtract 3% from the estimated annual hedge fund returns and calibrate our simulated time series accordingly. These simulated time series represent our first set of asset scenarios. We generate three additional sets of asset scenarios reflecting more negative expectations about the future hedge fund returns.

Mitchell and Pulvino (2001) find that one specific trading strategy used by hedge funds—the risk arbitrage strategy—shows zero correlation with the stock market during up-market conditions, but large positive correlation during down-market conditions. In this spirit, Agarwal and Naik (2004) examine a large number of hedge fund indices and find that these indices show no correlation in up-market conditions, but a positive correlation in down-market conditions, questioning the diversification benefit of hedge funds. Fung and Hsieh (2001), on the other hand, analyze the “trend-following” trading strategy applied by hedge funds and find that such hedge funds can substantially reduce the volatility of a typical stock and bond portfolio during extreme market downturns. To shed some light on the impact of different correlation structures on our results, we simulate a second set of asset scenarios based on correlation coefficients reflecting “bad” market conditions. More precisely, we sort the DAX and REXP returns and use only the below-median returns and the corresponding hedge fund returns to calculate correlation coefficients between the DAX and the different hedge fund indices as well as between the REXP and the different hedge fund indices. These correlation coefficients are presented in columns five and six of Table 1. Based on these correlations and the

\textsuperscript{20}Fung and Hsieh (2000), for example, find an instant history bias of 1.4% per year for hedge funds.

\textsuperscript{21}Another bias may occur in studies analyzing individual hedge fund performance characteristics. If the selected sample for such a study requires a minimum number of consecutive returns in the database, hedge funds with short survival times are systematically excluded. Fang and Hsieh (2000) call this phenomenon multiperiod sampling bias and conclude that this bias in estimated hedge fund returns is very small, if it exists at all. Baquero, ter Horst, and Verbeek (2005), however, split the hedge fund universe into ten deciles based on past performance, and report a multiperiod sampling bias of 3.8% for the worst-performing hedge funds.
previously specified parameter values, we derive a second set of asset scenarios.

The hedge fund return time series used in our analysis have very attractive characteristics. Even after accounting for the survivorship bias by reducing the hedge fund returns by three percent per annum, all hedge fund return series have higher Sharpe ratios than the DAX. Agarwal and Naik (2004) argue that the hedge fund performance observable during the last two decades is not representative for their long-term performance. They estimate a multifactor model of hedge fund returns. This model builds on excess returns of standard assets and options on these assets as risk factors. Using data for the years 1927 through 1989 for the exogenous variables, they are able to extrapolate hedge fund returns for this long time period. These generated returns are on average 2.76% lower than the mean annual returns during the 1990s, and the standard deviation is on average 2.57% higher compared to those of their recent performance. Reflecting these findings, we generate a third and a fourth set of asset scenarios reducing the mean hedge fund returns by 2.76% and increasing the standard deviation of hedge funds returns by 2.57%. All other parameters are as previously specified. The only difference between set number three and set number four is that we use the regularly estimated asset correlations (see columns 3 and 4 of Table 1) to generate set number three, and the correlations reflecting “bad” market conditions (see columns 5 and 6 of Table 1) to generate set number four.

With the adjustments made to the mean and standard deviation of the hedge fund returns in the third and fourth set of asset scenarios, five of the hedge fund indices have a Sharpe ratio that is lower than the Sharpe ratio of the DAX, and four hedge fund indices have a Sharpe ratio that is higher than that of the DAX. It is important to keep in mind that hedge fund returns are skewed and have a relatively larger kurtosis (see columns 7 and 8 of Table 1). Our asset model based on correlated AR(1) GARCH(1,1) processes adequately captures these characteristics for the purpose of our study; the null hypothesis that simulated and observed returns have the same distributions cannot be rejected by means of a Kolmogorov-Smirnov test and standard significance levels. However, we will return to this point when we discuss our results.

\[ \text{Sharpe ratio} = \frac{R - R_f}{\delta} \] assuming a yearly risk free rate \( R_f \) of 4% for convenience. The Sharpe ratio basically measures the excess return per unit of risk in an investment.
NUMERICAL RESULTS AND DISCUSSION

The Base Case

Let us first establish a reference case for our model insurance company with the parameter values specified in Table 3. We calculated two efficient frontiers for the insurance company. The first frontier is calculated allowing only investments in stocks and bonds. We then add hedge funds as a third investment alternative and calculate the second frontier. All calculations are based on 10,000 independent Monte Carlo simulations unless otherwise stated.

Figure 2 shows the two efficient frontiers with and without Convertible Arbitrage hedge funds as a possible investment opportunity. The difference between the four graphs in Figure 2 is that they are based on the four different sets of asset scenarios. Each of the graphs shows that the efficient frontier calculated with hedge funds as an investment alternative lies either above or at least on the one calculated without hedge funds for all four sets of scenarios. As expected, adding an additional investment alternative to the investment universe of our model insurance company helps to improve the investment performance of the company in some scenarios, without negative effects in the others.

Graph 1 of Figure 2 is based on the first set of asset scenarios; these scenarios are generated with the estimated parameter values. The asset allocations of these efficient portfolios include large hedge fund positions (up to 90.95%). The main reason for these high hedge fund percentages is that hedge funds reduce the overall volatility of the investment portfolio substantially. The hedge fund returns in this set of scenarios have a relatively low standard deviation on average. In addition, the correlation between hedge fund returns and stock market returns is relatively low and the correlation between hedge fund returns and bond returns is even negative in some cases (see Table 1). If we compare, for example, the standard deviations of the two efficient portfolios with 7.5% expected rate of return we find that the standard deviation of the portfolio with hedge funds is 3.28%, which is much lower than the 10.10% of the portfolio without hedge funds. The difference between the insurer’s shortfall probability is even more dramatic. A portfolio standard deviation of 10.10% in the case without hedge funds results in a shortfall probability of 27.69%. If the model insurer invests in hedge funds, however, it does not default in any one of the 10,000 scenarios. These results indicate that life insurance companies with cliquet-style interest rate guarantees should not hold volatile asset portfolios.
Graph 2 of Figure 2 presents the efficient frontiers for the second set of asset scenarios; these scenarios are generated with correlation coefficients based on below median returns. The efficient frontiers in Graph 2 hardly deviate from the ones in Graph 1. Therefore, we omit a detailed discussion of their characteristics.

Graph 3 of Figure 2 presents the efficient frontiers for our third set of asset scenarios; these scenarios are generated to reflect the long-term performance of hedge funds. Thus, in these scenarios, the mean hedge fund return is 2.76% lower and the standard deviation is 2.57% higher than in the first set of asset scenarios. The lowest shortfall probability of all efficient portfolios presented in Graph 3 is 0.49%. This minimum shortfall portfolio consists of 3.00% stocks, 80.69% bonds, and 16.31% hedge funds. The standard deviation of this portfolio is 2.96%. Overall, the hedge fund position in the efficient portfolios is now much smaller than in the ones calculated for the first set of asset scenarios. Therefore, we can conclude that the expected hedge fund return used in an asset model has a strong influence on the size of the optimal hedge fund position.

Table 3. Base Case Parameter Settings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_o$</td>
<td>20%</td>
</tr>
<tr>
<td>$g$</td>
<td>2.75%</td>
</tr>
<tr>
<td>$z$</td>
<td>5%</td>
</tr>
<tr>
<td>$a$</td>
<td>5%</td>
</tr>
<tr>
<td>$b$</td>
<td>30%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>90%</td>
</tr>
<tr>
<td>$d$</td>
<td>3%</td>
</tr>
<tr>
<td>$y$</td>
<td>50%</td>
</tr>
<tr>
<td>$T$</td>
<td>10</td>
</tr>
</tbody>
</table>

Initial reserve quota of the insurance company at time $t = 0$.
Minimum interest rate guarantee.
Target interest rate. The insurance company plans to credit $z$ to the policyholders’ accounts every year.
Trigger for management intervention. If the reserve quota $x_t < a$, the management cuts the target interest rate and takes other actions to bring $x_t$ within the pre-specified range $[a, b]$.
Trigger for management intervention. The management intervenes if $x_t > b$.
Minimum participation rate. At least the fraction $\delta$ of the investment returns on a book value basis have to be credited to the policyholders’ accounts.
Dividend factor.
Accounting system parameter. The fraction $y$ of rises in the market value of assets is reflected in the book values.
Number of time periods (years) considered.

Graph 4 of Figure 2 presents the efficient frontiers for the fourth set of asset scenarios; these scenarios are generated with mean hedge fund returns reduced by 2.76%, standard deviation of hedge fund returns increased by 2.57%, and correlation coefficients based on below-median
returns. The efficient frontiers in Graph 4 hardly deviate from the ones in Graph 3. Therefore, we omit a detailed discussion of their characteristics.

Let us now examine how different types of hedge funds influence the efficient portfolios of our model life insurer. We assume that either due to regulatory constraints or based on a management decision the insurance company sets a 5% limit for hedge funds in its asset allocation.\(^{23}\) Given this constraint, we calculate the efficient frontiers for all nine hedge fund styles. The actual ranking of the hedge fund styles according to their expected

\(^{23}\)More precisely, the 5% limit is imposed on the fraction \(q_3\) of hedge funds in the asset allocation. Hence, at \(t = 1\) the insurer cannot invest more than 5% of its assets in hedge funds. In later time periods, however, the percentage of hedge funds in the investment portfolio also depends on the development of the price processes \(S_{1,t}, S_{2,t}\) and \(S_{3,t}\). Since the model does not allow rebalancing, it is possible that the fraction of hedge funds in the investment portfolio exceeds 5% in later time periods.
rates of return for a given shortfall probability of 5% is provided in the Appendix. Using the asset scenario sets three and four, the optimal hedge fund holding for a given shortfall probability of 5% is zero for some hedge fund styles. To ensure comparability across the four sets of asset scenarios, we set the hedge fund holding equal to 5% in the optimizations for asset scenario sets three and four. We find that the Event-Driven Multi-Strategy hedge funds perform best for all asset scenarios, and the Global Macro hedge funds perform worst for three of the asset scenario sets. Most rankings are relatively stable across the different sets of asset scenarios. The difference in the expected rate of return of the two extreme cases is between 0.42% and 0.32%, depending on the asset scenarios used.

The Liability Structure’s Influence on Efficient Portfolios

We now vary some of the parameters modeling the insurer’s liability structure and examine the effect of these parameter variations on the insurer’s incentives to invest in hedge funds.

In a first step, we vary the guaranteed rate of interest and assume that our model insurance company guarantees $g = 4\%$ instead of $g = 2.75\%$. Graph 1 of Figure 3 shows the efficient frontiers with and without hedge funds for the base case as well as for the case with a 4% interest rate guarantee. In both cases, the efficient frontier with hedge funds lies above the one calculated without hedge funds. If we examine the asset allocations of the efficient portfolios for the case of the higher guarantee we find that they include a large hedge fund position (up to 97.58%). Comparing the efficient portfolios in this scenario with the base case, we find that the fraction of hedge funds is higher for insurance companies with higher interest rate guarantees. The intuition behind this result is that companies with higher interest rate guarantees should further reduce the volatility of their asset portfolios. However, the most interesting result is that the difference between the efficient frontiers with and without hedge funds is greater for insurers with higher interest rate guarantees. Therefore, such insurance companies can gain more from adding hedge funds to their investment universe and, hence, have a greater incentive to invest in hedge funds. This incentive difference does not seem to exist when we examine the adjusted asset scenarios reflecting long-term hedge fund performance (see Graph 1 of Figure 4). The other difference between the standard and the adjusted asset scenarios is that for the adjusted scenarios the hedge fund position in efficient portfolios is smaller for insurance companies with higher interest rate guarantees than in the base case. More precisely, the maximum hedge fund holding in efficient portfolios is 13.09% for our model insurance company with the higher guaranteed interest rate and 20.50% for the reference case.
Second, we reduce the insurance company’s initial reserve quota \( x_0 \) to 10\% instead of the 20\% in the base case. This means that our model insurance company is now much less capitalized. Graph 2 of Figure 3 shows the efficient frontiers with and without hedge funds for the less-capitalized insurance company as well as for the reference company. Once again, for the less-capitalized insurer, the efficient frontier with hedge funds lies above the one calculated without hedge funds. For a shortfall probability above 0.1\%, the fraction of hedge funds in efficient portfolios of the less capitalized insurer is higher than in the base case. We also find that the difference between the efficient frontiers with and without hedge funds is greater for less-capitalized insurers. In other words, less-capitalized insurers can gain more from investing in hedge funds and, hence, have a greater incentive to do it. This important result also holds if we analyze the adjusted asset scenarios reflecting the lower long-term hedge fund performance (see Graph 2 of Figure 4). However, for the adjusted scenarios,
the hedge fund position in efficient portfolios is smaller for less-capitalized insurance companies than for their well-capitalized peers. More precisely, the maximum hedge fund holding in efficient portfolios is 13.09% for the less-capitalized model insurance company and 20.50% for the base case.

Third, we vary the target interest rate and assume that our model insurance company plans to credit $z = 6.5\%$ instead of $z = 5\%$ to the policyholders’ accounts every year. Since consumers are price sensitive, they pay close attention to interest paid by insurance companies. Thus, promising a high interest rate can be interpreted as an aggressive management strategy seeking additional market share. Graph 3 of Figure 3 shows the efficient frontiers with and without hedge funds for the more aggressive insurance company as well as for the reference company.

The efficient frontier with hedge funds lies above the one calculated without hedge funds. For all shortfall probabilities above 0.2%, the fraction of hedge funds in efficient portfolios of the more aggressive insurer is

Fig. 4. Efficient frontiers for different parameter settings—asset scenarios adjusted to reflect long-term hedge fund performance.
higher than in the base case. We also find that the difference between the efficient frontiers with and without hedge funds is greater for insurance companies with a higher target interest rate. In other words, insurance companies with an aggressive management strategy can gain more from adding hedge funds to their investment universe and, hence, have a greater incentive to invest in hedge funds. This important result holds as well if we analyze the adjusted asset scenarios reflecting the lower long-term hedge fund performance (see Graph 3 of Figure 4). However, for the adjusted scenarios, the hedge fund position in efficient portfolios is smaller for the more aggressive insurance company than for the reference company. More precisely, the maximum hedge fund holding in efficient portfolios is 12.30% for our model insurance company with the higher target interest rate and 20.50% for the base case.

Fourth, we analyze how a change in the accounting system toward a full fair value–oriented accounting approach affects an insurance company’s liability structure and how the resulting structure affects the company’s decisions to invest in hedge funds. By setting the accounting system parameter in our model to $y = 0.95$, our model insurer is forced to show 95% of all rises in the market value of its assets on its balance sheet. Such a “close to full fair value” accounting system restricts the possibilities of the management to redirect investment gains into hidden reserves, and, hence, reduces the financial buffer of the insurance company. Note that $y = 0.5$ in the base case. Graph 4 of Figure 3 shows the efficient frontiers with and without hedge funds for the two different accounting environments. Let us focus on the full fair value accounting case first. The efficient frontier with hedge funds lies above the one calculated without hedge funds. For all shortfall probabilities above 0.2%, the fraction of hedge funds in efficient portfolios of an insurer operating in a full fair value accounting environment is higher than in the base case. Analogous to the previous cases, we find that the difference between the efficient frontiers with and without hedge funds is greater for insurer companies in a fair value accounting environment. In other words, such an environment creates greater incentives to invest in hedge funds. This important result holds as well in the analysis using the adjusted asset scenarios (see Graph 4 of Figure 4). However, for the adjusted scenarios, the hedge fund position in efficient portfolios is smaller for insurance companies operating in a full fair value accounting environment, which does not allow for the creation of hidden reserves, than in the base case. More precisely, the maximum hedge fund holding in efficient portfolios is 14.30% for our model insurance company operating in the full fair value accounting environment and 20.50% for the base case.
In summary, insurance companies with higher interest rate guarantees, lower financial reserves, and plans to credit higher target interest rates to the policyholders’ accounts, as well as insurance companies operating in a full fair value accounting environment, have a greater incentive to invest in hedge funds. Adding hedge funds as a possible investment alternative increases the expected portfolio return for these insurance companies much more than for comparable companies with lower interest rate guarantees, higher financial reserves, or a lower target interest rate, or companies operating in a more conservative accounting system, which offers possibilities to smooth reported investment earnings. On the other hand, those insurance companies with higher incentives to invest in hedge funds should not necessarily invest more in hedge funds than their counterparts. If future hedge fund returns have the same characteristics as those returns reported since the 1990s, then insurers with higher guarantees, higher target interest rates, and lower reserves should invest a greater proportion of their assets in hedge funds than their counterparts. This result is especially important for insurance regulators. Higher interest rate guarantees as well as high target interest rates increase the liabilities of an insurance company, and, hence, *ceteris paribus*, decrease the financial strength of the company. Therefore, our results show that financially weak insurers have stronger incentives to invest in hedge funds than their well-capitalized counterparts. According to a model calibrated with recent hedge fund return data, their hedge fund position should be quite high. But a more conservative model calibration reflecting the lower long-term performance predicted by Agarwal and Naik (2004) suggests a much smaller hedge fund holding for such weak insurance companies. Furthermore, such weak insurers probably do not have the capital to deal with extreme situations such as high insurance claims and a simultaneous market downturn, which reveals a negative change in hedge fund return characteristics; and a huge hedge fund holding might exacerbate their situation. Moral hazard could encourage very weak insurers to increase their hedge fund holdings substantially in a “go-for-broke” behavior. Thus, insurance regulators should monitor the hedge fund positions of financially weak insurers closely.

**DISCUSSION OF RESULTS**

We use correlated AR(1) GARCH(1,1) processes to model returns of stocks, bonds, and hedge funds. While it would be possible to use different stochastic processes for the three investment alternatives, we follow previous literature and adopt a more parsimonious modeling approach (e.g.,
Amin and Kat, 2002; Cvitanić, Lazrak, Martellini, and Zapatero, 2003; Popova, Morton, and Popova, 2003; Terhaar, Staub, and Singer, 2003; Boyle and Liew, 2007). We use the same parametric stochastic process for all assets with customized parameter values for each asset. We think AR(1) GARCH(1,1) processes capture sufficient salient details for the purposes of our study. When testing for differences between simulated returns and returns of the German stock index DAX, the German bond index REXP, and the CASAM CISDM hedge fund strategy indices with a Kolmogorov-Smirnov test, we do not find significant distributional differences.

While we use a number of adjustments for the parameters of the model to reflect observed hedge fund return characteristics, it is still possible that the model does not capture the inherent extreme tail risk of the hedge fund returns. An alternative model framework that explicitly focuses on “extreme states of the world” is the regime-switching model. Boyle and Liew (2007) use a regime-switching model to examine optimal portfolios of a utility-maximizing investor with hedge funds on the menu. The investor chooses which fractions of initial wealth to invest in a hedge fund index, the S&P 500 index, and the risk free rate. One of Boyle and Liew’s main findings is that the hedge fund weight in the optimal portfolio increases with the investor’s level of risk aversion. Our results demonstrate that weakly capitalized insurers and insurers providing higher interest rate guarantees have higher incentives to invest in hedge funds than financially strong insurers and insurers providing relatively low guarantees. Hence, our findings for insurance companies complement Boyle and Liew’s results for utility-maximizing investors.

Although Boyle and Liew (2007) use an asset model that explicitly captures left-tail risk, they acknowledge in their discussion that their expected utility-maximization approach to form asset portfolios does not focus on left-tail risk. They further highlight that including left-tail risk aversion into investors’ utility functions has an impact on the fraction of hedge funds in optimal portfolios. While our construction of investment portfolios based on an insurer’s shortfall probability does a better job of incorporating left-tail risk than expected utility maximization, our hedge fund return model does not focus on extreme tail risk. Therefore, our results with respect to the exact fraction of hedge funds in an insurer’s efficient investment portfolio as well as the rankings of hedge fund styles presented in Table 5 can be viewed as being model specific. Using a regime-switching hedge fund return model is likely to reduce the fraction of hedge funds in

We would like to thank an anonymous reviewer for highlighting this issue.
efficient portfolios. A detailed analysis of optimal hedge fund positions in insurers’ investment portfolios based on a variety of asset return models and/or alternative risk measures is beyond the scope of this paper, but provides an interesting topic for future research.

A related point is that we use indices of hedge fund returns and not returns of individual hedge funds to calibrate our model. When forming an index of securities, one reduces the variance even if the securities are positively correlated. Furthermore, the impact of aggregation on return characteristics is more pronounced in the tail of the distribution since a given fund may do very poorly or fail. Using returns of an individual hedge fund rather than an index should reduce the fraction of hedge funds in efficient portfolios.

However, our main result that the strong incentives for financially weak insurers might create a moral hazard to overinvest in hedge funds should also hold under a regime-switching model. If the starting regime is the “good-capital-markets state of the world,” very weak insurers might even have stronger incentives to overinvest in hedge funds than in our AR(1) GARCH(1,1) model. For insurers close to financial distress, a bet on not having a capital market regime switch may look very lucrative. Hence, insurance regulators should especially monitor the hedge fund positions of financially weak insurers in bullish capital market periods.

An important characteristic of many hedge funds is that they have a variety of features that make them less liquid than investing in an equity portfolio. First, a prudent investor in a hedge fund should conduct due diligence. This can be a costly process, and in the post-Madoff era we see that even institutional investors can be fooled for a long time. Second, most hedge funds have share restrictions like lockup periods, notice periods, and redemption periods. During the lockup period an investor cannot withdraw funds. After the lockup period, an investor who wants to withdraw has to give advance notice and wait some additional time to receive the money. Such share restrictions give investment managers more discretion, and Agarwal, Daniel, and Naik (2009) find that managerial discretion proxied by longer lockup, notice, and redemption periods is positively related to hedge fund performance. Joenväärä and Tolonen (2009) and de Roon, Guo, and ter Horst (2009), on the other hand, dispute this finding. Third, in periods of financial crises, hedge fund investors may be faced with an additional liquidity problem: the price impact of the hedge fund’s fire sales. To fund withdrawals of impatient or cash strapped investors, hedge fund managers may have to quickly sell securities at depressed values. This very action of adding to the supply may serve to further

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25We thank an anonymous reviewer for highlighting this issue.
depress prices and, hence, the value of the hedge fund’s remaining securities holdings. It is possible to explicitly model illiquidity of hedge funds (e.g., de Roon, Guo, and ter Horst, 2009) and to incorporate illiquidity considerations in an asset-liability-management model of an insurance company (e.g., Berry-Stölzle, 2008a, 2008b). However, this is outside the scope of this paper.

Caveats

By design, our study is based on a simple model of asset returns. Hence, we would like to highlight that any interpretation of our results should keep the following caveats in mind. First, the AR(1) GARCH(1,1) process used to model hedge fund returns may not completely capture the extreme tail risk inherent in hedge fund returns. While we think an AR(1) GARCH(1,1) process captures the relevant return characteristics for the purpose of our study, analyzing jump processes or other stochastic asset models that explicitly incorporate extreme tail events provides an interesting route for future research (see also Discussion of Results section). Second, the model framework does not explicitly model the illiquidity of hedge fund investments. While it is standard to ignore liquidity considerations in investment portfolio models (e.g., Amin and Kat, 2002; Cvitanić, Lazrak, Martellini, and Zapatero, 2003; Popova, Morton, and Popova, 2003; Boyle and Liew, 2007), future research may incorporate hedge fund illiquidity using an approach similar to Berry-Stölzle (2008a) or Berry-Stölzle (2008b) (see also Discussion of Results Section). Third, we use indices of hedge fund returns to calibrate the model. This is a standard approach in investment portfolio models (see, e.g., Boyle and Liew, 2007), yet indices based on hedge fund databases suffer from a number of biases including the survivorship bias and the bias caused by hedge funds’ self-selection of reporting to the databases. While we address biases inherent in hedge fund databases by subtracting 3% from the estimated annual hedge fund returns (see, e.g., Popova, Morton, and Popova, 2003, for a similar approach), this adjustment of the hedge fund return mean does not capture variations in the tail of the return distribution (see also Data and Model Calibration section). Future research should analyze return data of individual hedge funds in the context of an asset-liability-management model of an insurance company.

CONCLUSION

In this paper we study the impact of hedge fund investments on the risk exposure of a life insurance company selling with-profit life insurance
policies with a cliquet-style interest rate guarantee. We address specific characteristics of hedge fund returns in our Monte Carlo simulation analysis with a three-dimensional AR(1) GARCH (1,1) asset model. We calibrate our model to the German stock index (DAX), the German bond index (REXP), and the hedge fund indices of the Center for International Securities and Derivatives Markets (CISDM), and correct the estimated hedge fund returns for biases inherent in self-reported hedge fund return data. We then use the estimated parameters as well as three additional sets of parameters reflecting more negative expectations about future hedge fund returns in our simulations.

Our analysis provides the following main results: First, the higher the interest rate guarantee is, the less capital an insurer holds, the higher the interest rate is that an insurer actually plans to credit to its policyholders’ accounts, and the fewer possibilities the accounting system provides to smooth the book values of the investment portfolio, the more incentives an insurance company has to diversify its portfolio by investing in hedge funds. Second, any advantage to the amount of hedge fund investment for an individual relative to other insurers is dependent on the expectation about future hedge fund returns. Those insurers with higher incentives to invest in hedge funds should invest a bigger fraction in hedge funds than their counterparts only if future hedge fund returns will have the same favorable characteristics as the observed returns since the 1990s. If future hedge fund returns will be lower, those insurers should invest less in hedge funds than their peers.

The fact that weakly capitalized insurers have a strong incentive to invest in hedge funds, but their hedge fund holdings should be lower than the optimal hedge fund holdings of well-capitalized insurers if one uses conservative expectations about future hedge fund returns, should especially be of interest to insurance regulators. Moral hazard could encourage very weak insurers to increase their hedge fund holdings substantially in a “go-for-broke” behavior. In a “bull-market” for hedge funds, such a huge hedge fund holding might generate high returns, but these insurers might not be prepared for unfavorable regime changes or an extreme market crash.

REFERENCES


Amin, GS and HM Kat, 2002, Stocks, Bonds, and Hedge Funds: Not a Free Lunch, ICMA Centre Discussion Papers in Finance, University of Reading.


## Appendix Table 1. Hedge Fund Styles—Classification Based on the CASAM CISDM Hedge Fund Strategy Indices

<table>
<thead>
<tr>
<th>Strategy group</th>
<th>Type of hedge fund strategy</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative value</td>
<td>Relative value Equity market neutral index</td>
<td>Take long positions in undervalued equities and offsetting short positions in over-valued equities within the context of minimizing the market exposure in the underlying equity market traded.</td>
</tr>
<tr>
<td></td>
<td>Convertible arbitrage index</td>
<td>Take long positions in convertible securities and try to hedge those positions by selling short the underlying common stock. The goal is to minimize the market exposure.</td>
</tr>
<tr>
<td>Event-driven</td>
<td>Distressed securities index</td>
<td>Take positions in the securities of companies where the price has been or is expected to be affected by a distressed situation, such as an announcement of reorganization due to financial or business difficulties. The goal is to make use of event-driven return opportunities that may be independent of general market movements.</td>
</tr>
<tr>
<td></td>
<td>Event-driven multi-strategy index</td>
<td>Attempt to predict the outcome of corporate events like liquidations, spin-offs, and other transactions and take the necessary position to make a profit. The goal is to make use of event-driven return opportunities that may be independent of general market movements.</td>
</tr>
<tr>
<td></td>
<td>Merger arbitrage Index</td>
<td>Concentrate on companies that are the subject of mergers, tender offers, or exchange offers in order to use event-driven return opportunities.</td>
</tr>
<tr>
<td>Opportunistic</td>
<td>Emerging markets index</td>
<td>Invest in equities or sovereign debt of companies located in emerging or developing economies. The underlying market exposure will not be systematically eliminated or minimized.</td>
</tr>
<tr>
<td></td>
<td>Equity long/short index</td>
<td>Take long and short equity positions varying from net long to net short, depending on whether the market is bullish or bearish. No systematic elimination or minimization of the underlying market exposure.</td>
</tr>
<tr>
<td></td>
<td>Global macro Index</td>
<td>Employ opportunistically long and short multiple financial and non-financial assets using systematic trend-following models. No systematic elimination or minimization of the underlying market exposure.</td>
</tr>
<tr>
<td>Total</td>
<td>Equal weighted hedge fund index</td>
<td>Represents the overall composition of the hedge fund universe.</td>
</tr>
</tbody>
</table>

*Note:* This classification of hedge fund styles is based on the CASAM CISDM hedge fund strategy indices from the Center for International Securities and Derivatives Markets (CISDM). *Source:* CISDM.
## Appendix Table 2. Ranking of Hedge Fund Styles

<table>
<thead>
<tr>
<th>Rank</th>
<th>Hedge fund style</th>
<th>Rate of return (in%)</th>
<th>Asset allocation (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>DAX</td>
<td>REXP</td>
</tr>
<tr>
<td>A. Scenarios based on estimated parameter values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Event-driven</td>
<td>7.082</td>
<td>24.09</td>
</tr>
<tr>
<td>2</td>
<td>Equal weighted</td>
<td>7.037</td>
<td>21.85</td>
</tr>
<tr>
<td>3</td>
<td>Convertible arbitrage</td>
<td>6.910</td>
<td>19.97</td>
</tr>
<tr>
<td>4</td>
<td>Merger arbitrage</td>
<td>6.891</td>
<td>23.80</td>
</tr>
<tr>
<td>5</td>
<td>Equity long/short</td>
<td>6.865</td>
<td>23.55</td>
</tr>
<tr>
<td>6</td>
<td>Distressed securities</td>
<td>6.858</td>
<td>20.28</td>
</tr>
<tr>
<td>7</td>
<td>Equity market neutral</td>
<td>6.707</td>
<td>17.88</td>
</tr>
<tr>
<td>8</td>
<td>Global macro</td>
<td>6.701</td>
<td>21.63</td>
</tr>
<tr>
<td>9</td>
<td>Emerging markets</td>
<td>6.665</td>
<td>19.46</td>
</tr>
<tr>
<td>B. Return correlations based only on below-median returns</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Event-driven</td>
<td>7.076</td>
<td>23.97</td>
</tr>
<tr>
<td>2</td>
<td>Equal weighted</td>
<td>7.032</td>
<td>21.74</td>
</tr>
<tr>
<td>3</td>
<td>Equity long/short</td>
<td>6.907</td>
<td>19.88</td>
</tr>
<tr>
<td>4</td>
<td>Merger arbitrage</td>
<td>6.878</td>
<td>23.57</td>
</tr>
<tr>
<td>5</td>
<td>Convertible arbitrage</td>
<td>6.868</td>
<td>23.58</td>
</tr>
<tr>
<td>6</td>
<td>Distressed securities</td>
<td>6.856</td>
<td>20.21</td>
</tr>
<tr>
<td>7</td>
<td>Equity market neutral</td>
<td>6.708</td>
<td>21.73</td>
</tr>
<tr>
<td>8</td>
<td>Global macro</td>
<td>6.697</td>
<td>19.93</td>
</tr>
<tr>
<td>9</td>
<td>Emerging markets</td>
<td>6.663</td>
<td>17.27</td>
</tr>
<tr>
<td>C. Hedge fund returns adjusted to reflect long-term performance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Event-driven</td>
<td>6.801</td>
<td>22.13</td>
</tr>
<tr>
<td>2</td>
<td>Equal weighted</td>
<td>6.748</td>
<td>19.97</td>
</tr>
<tr>
<td>3</td>
<td>Merger arbitrage</td>
<td>6.660</td>
<td>22.10</td>
</tr>
<tr>
<td>4</td>
<td>Equity long/short</td>
<td>6.648</td>
<td>18.46</td>
</tr>
<tr>
<td>5</td>
<td>Convertible arbitrage</td>
<td>6.648</td>
<td>22.03</td>
</tr>
<tr>
<td>6</td>
<td>Distressed securities</td>
<td>6.602</td>
<td>18.68</td>
</tr>
<tr>
<td>7</td>
<td>Emerging markets</td>
<td>6.508</td>
<td>16.85</td>
</tr>
<tr>
<td>8</td>
<td>Equity market neutral</td>
<td>6.485</td>
<td>19.94</td>
</tr>
<tr>
<td>9</td>
<td>Global macro</td>
<td>6.449</td>
<td>18.06</td>
</tr>
<tr>
<td>D. Hedge fund returns as in C and asset correlations as in B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Event-driven</td>
<td>6.782</td>
<td>21.78</td>
</tr>
<tr>
<td>2</td>
<td>Equal weighted</td>
<td>6.739</td>
<td>19.80</td>
</tr>
<tr>
<td>3</td>
<td>Convertible arbitrage</td>
<td>6.658</td>
<td>22.18</td>
</tr>
<tr>
<td>4</td>
<td>Merger arbitrage</td>
<td>6.646</td>
<td>21.87</td>
</tr>
<tr>
<td>5</td>
<td>Equity long/short</td>
<td>6.640</td>
<td>18.29</td>
</tr>
<tr>
<td>6</td>
<td>Distressed securities</td>
<td>6.599</td>
<td>18.59</td>
</tr>
<tr>
<td>7</td>
<td>Equity market neutral</td>
<td>6.491</td>
<td>20.03</td>
</tr>
<tr>
<td>8</td>
<td>Global macro</td>
<td>6.471</td>
<td>18.32</td>
</tr>
<tr>
<td>9</td>
<td>Emerging markets</td>
<td>6.461</td>
<td>16.20</td>
</tr>
</tbody>
</table>