Pricing of Deposit Insurance Considering Investment, Deductibles, and Policy Limit

Hong Mao,1 Krzysztof M. Ostaszewski,2 James M. Carson,3 and Yuling Wang4

Abstract: We discuss a risk-based valuation model for deposit insurance. Our model considers the investment activities of the deposit insurer, deductibles, and regulatory capital requirements to determine the premium rate for deposit insurance. To mitigate the moral hazard problem inherent in deposit insurance, we propose an upper limit for claim payments that is a decreasing function of a bank’s insolvency probability. We discuss time-varying default risk in the context of the deposit insurance pricing model. [Key words: insolvency, deposit insurance, moral hazard, market discipline, risk-based premiums.]

INTRODUCTION

The topic of deposit insurance for banking institutions has attracted much attention in the literature. Vanhoose (2007) reviews theories of bank behaviors under capital regulation considering moral hazard and the role of deposit insurance. It is well known that deposit insurance results in moral hazard and encourages insured banks to take more risk. Various methods to decrease moral hazard and to reduce the default probability of insured banks have been proposed, including applying minimum capital requirements, setting caps on the claim payments by the FDIC, and fairly pricing deposit insurance.5

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The main concern in the fair pricing of deposit insurance is how to make the premium properly reflect the risk of the insured bank (Hwang et al., 2009). Pioneering work by Merton (1977) proposes modeling deposit insurance as a put option, and Merton’s work has been extended by Ronn and Verna (1986), Thomson (1987), Episcopos (2004), and others.

Kaufman (2012) notes, “Over time the insurance-induced weakening of depositor discipline over banks caused a mostly unnoticed weakening of the financial condition of individual banks.” In other words, the discipline-inducing role of depositors and creditors in general has been effectively diminished by deposit insurance. Calomiris (2009) shows how this banking structure has led to unsound risk-taking activities by banks, largely manifesting in what has come to be known as the Great Recession. Our work suggests that small adjustments to contract structures that appear harmful to depositors (e.g., policy limits and deductibles) can improve depositor incentives to monitor the insurer.

The issue of the investment policy of the deposit insurer largely has been ignored in the existing literature, yet the case can be made that the insurer’s investment activities should relate to the premium rate of deposit insurance. Just as in the case of general insurance, some allocation to stocks may be optimal, and investment policy should be determined by risk management policy. If investment in stocks is indeed too risky, then an optimal portfolio will contain no stocks. The return earned by the deposit insurer can offset at least a portion of the default risk of the insured banks, and thus influence the deposit insurance premium. But the selection of investment portfolio for the deposit insurer must be appropriately conservative, and potentially hedge the risks faced by the depositors of the insured banks. For example, FDIC’s investment policy is highly regulated, with a view for the safety and liquidity of the funds. In our work, we establish general models to determine an optimal investment portfolio of deposit funds based on the safety-first criterion (Roy, 1952).

Providing deposit insurance creates moral hazard. For many types of insurance, straight deductibles and policy limits are standard methods of controlling the moral hazard problem (see Borch, 1975). Arrow (1971) shows that, for a fairly-priced contract, a risk-averse agent prefers full insurance above a deductible. As noted by Cummins and Mahul (2004),

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5For a discussion of minimum capital requirements, see Cooper and Ross (2002). Duffie et al. (2003) discuss deposit insurance for the U.S. and the role of the Federal Deposit Insurance Corporation (FDIC). Cummins (1986) develops risk-based premiums related to guaranty funds for insurers.

6In addition, shareholders own bank shares within diversified portfolios, and such dispersed ownership structure reduces their incentives for bank monitoring.
this deductible equals zero for insurance having an actuarially fair price and no policy limit. However, considering other assumptions, such as a limited budget, an optimal insurance contract includes a policy limit.

In this paper, we discuss the pricing of deposit insurance with consideration of deductibles and policy limits. We propose deposit insurance with a policy limit that is a decreasing function of bank insolvency probability, as opposed to a fixed limit, in order to encourage banks to reduce risk-taking behavior and to provide a method of determination of the premium rate in this case. Further, we examine how the capital adequacy ratio, the standard deviation of the stochastic rate of return on bank assets, and deductibles affect the deposit insurance premium rate. In this work we model and analyze the behavior of both insured banks and the provider of deposit insurance and their mutual influence on each other in determining the premium rate of deposit insurance. The intuition behind our model is that the presence of insurance results in moral hazard, but the design and pricing of the insurance product can be used as effective remedies for the potentially significant moral hazard problem.

Recent developments in deposit insurance in the U.S. potentially call for pricing models that are more dynamic and that are able to accommodate time-varying default risk. In this paper, we discuss how to establish such a pricing model of pricing deposit insurance that considers a dynamic default rate.

The following section presents the models of deposit insurance pricing. Then, the third section provides a sensitivity analysis among various related factors. The fourth section provides a discussion of pricing deposit insurance when considering the dynamic default rate, and the final section summarizes and concludes the paper.

**THE MODELS FOR DEPOSIT INSURANCE PRICING**

**Assumptions**

Assume that the initial deposit of the bank is equal to 1 and that the initial equity capital as a proportion of the deposit is \( c \), also called capital adequacy rate. Therefore, the bank has \( 1 + c \) available at the beginning of the period. Assume that the default probability of the bank is \( Q \), the recovery value per amount lent on the default loans is equal to \( k \), the deductible amount of the deposit insurance policy is \( d \), and the average interest rate of the deposit is \( r_B \). Assume that the return rate of investment portfolio for the deposit insurance funds can be expressed as the following stochastic differential equation\(^7\):
\[ dr = \beta(\mu_r - r)dt + \sigma_r dz_r. \] (1)

Assuming that the proportion of i-th investment is \( \alpha_i, i = 1, 2, \ldots, n \) and \( \sum_{i=1}^{n} \alpha_i = 1 \), then the return on risky asset i follows equation (2):

\[ dr_i = \beta_i(\mu_i - r_i)dt + \sigma_i dz_i \] (2)

and the portfolio return is \( \sum_{i=1}^{n} \alpha_i r_i \). The differential of the portfolio return is:

\[ \sum_{i=1}^{n} \alpha_i dr_i = \sum_{i=1}^{n} \alpha_i(\beta_i(\mu_i - r_i)dt + \sigma_i dz_i) = \] (3)

\[ \sum_{i=1}^{n} \alpha_i \beta_i(\mu_i - r_i)dt + \sum_{i=1}^{n} \alpha_i \sigma_i dz_i. \]

If \( \beta_i = \beta \), then \( \mu_r = \sum_{i=1}^{n} \alpha_i \mu_i \), otherwise, we let

\[ \beta = \frac{1}{n} \sum_{i=1}^{n} \beta_i, \text{then } \mu_r \approx \sum_{i=1}^{n} \alpha_i \mu_i. \] (4)

If the correlation between \( z_i \) and \( z_j \) is \( \rho_{ij} \) and \( \sigma_i \) and \( \sigma_j \) are the standard deviations of the i-th investment and the j-th investment, then the variance of the portfolio return is

\[ V_p \left( \sum_{i=1}^{n} \alpha_i \sigma_i dz_i \right) = \left( \sum_{i=1}^{n} \alpha_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij} \right) dt \]

and the standard deviation is

\footnote{Since the return rate can be either positive or negative, we employ the Vasicek (1977) model. While equity returns often are modeled with a generalized Wiener process, we use one model, the Vasicek model, for bond and equity returns, as the model is adaptable to our analysis and allows us to focus on the key issues of the paper.}
where $dz_r$ is a standard Wiener process, $\sigma_r$ is the standard deviation of the return rate for the investment portfolio, $\mu_r$ is the long-term equilibrium investment rate of return for the portfolio, and $\beta$ is the speed at which the rate of return of the investment portfolio returns to its long-term equilibrium level.

### The Selection of Optimal Investment Portfolio

Bond and Crocker (1993) discuss the problem of investment by the deposit insurer. They assume that the investment return rate is a constant, and do not consider the effects of different strategies of investment portfolios. Although FDIC invests deposit insurance funds only in Treasury bills, here we assume that banks may adopt equity positions for a portion of these funds, keeping in mind the objective of investment safety. Here we evaluate stochastic investment return rates on the different investment portfolios. As the premium income obtained by the deposit insurer creates a liability, and its function is to provide insured banks with insurance protection, the selection of investment portfolio should be based on an appropriate risk management policy, such as the “Safety First” criterion (Roy, 1952) that we incorporate here. With this consideration, we establish the following objective function:

$$
\text{Min} \ p = \text{Pr}(r < r_L | t \leq T).
$$

Here, $p$ is the probability that the investment rate of return $r$ is less than $r_L$, the minimum level of the return rate of investment acceptable by the investor, and $T$ is maturity time of deposit insurance.

Using Monte Carlo simulation, we find the optimal solution of $E(r^*)$, $p^*$, and the optimal proportions of investment portfolio, $\alpha_i^*$, for $i = 1,2,\ldots,n$. The optimization process is described as follows: Considering equation (1) as

$$
\Delta r = r_t - r_{t-1},
$$

$$
r_t = r_{t-1} + \beta(\mu_r - r_{t-1})\Delta t + \sigma_r \varepsilon \sqrt{\Delta t},
$$

we produce random number $\varepsilon$, which follows standard normal distribution. Let $\Delta t = 1$ and calculate the value of $r_t, t = 1,2,\ldots,T$ (for each time $t$, (5)
10,000 paths are simulated). For each pair of \(\alpha_1, \alpha_2\) taking the values from 0 to 1, step = 0.01 and combine between each other; then \(\alpha_3 = 1 - \alpha_1 - \alpha_2\), and we count the number satisfying \(r < r_L\) under the condition of \(t \leq T\). We find the approximated value of the probability of \(r < r_L\), \(p = \Pr(r_t < r_L, t \leq T)\) and take the values of \(\alpha_1, \alpha_2\) and \(1 - \alpha_1 - \alpha_2\) which satisfy the objective of \(\min p = \Pr(r_t < r_L, t \leq T)\) as the optimal proportions of the investment portfolio.

We assume that the investment portfolio includes three asset classes: U.S. stocks, U.S. Treasury bonds, and U.S. Treasury bills. The portfolio allocations for each asset class are \(\alpha_1, \alpha_2,\) and \(\alpha_3\), respectively. We use S&P 500 stock market index data and rates of return for Treasury bonds and for Treasury bills. For these three asset classes and the period 1965 to 2008, we estimate means, standard deviations, and correlation coefficients, and the values of \(\beta, i = 1, 2, 3\).

The results are as follows:

\[
\begin{align*}
[\mu_1, \mu_2, \mu_3] &= [0.1048, 0.0767, 0.0577] \\
[\sigma_1, \sigma_2, \sigma_3] &= [0.1776, 0.0937, 0.0277]
\end{align*}
\]

and

\[
\rho = \begin{bmatrix}
1 & 0.144 & 0.1322 \\
0.144 & 1 & 0.115 \\
0.1322 & 0.1115 & 1
\end{bmatrix}.
\]

We assume \(r_L = 0\), the initial return rate of investment \(r_0 = 0.05\), and \(\beta = 0.10\). We use a two-period (\(T = 2\)) stochastic model in order to simplify the calculation. Then the optimal investment portfolio that is satisfied with objective function \(\min p = \Pr(r_t \leq r_L, t \leq T)\) is \([\alpha_1^*, \alpha_2^*, \alpha_3^*] = [0.05, 0.10, 0.85]\) and the optimal solution is \(p^* = 0.0765\) and \(E(r^*) = 0.0534\).

The Pricing Model with Deductibles

Deductibles are widely used in various types of insurance. Schlesinger (1981) explores, among other issues related to deductibles, the conditions under which full coverage or no coverage may be optimal. As discussed by Gollier (1996), use of deductibles often is the “best compromise between the desire to cover large losses and the willingness to limit the cost of insurance” (p. 369). Thus, we examine deductibles in the context of deposit insurance. The objective for our use of a deductible is to formulate an incentive-compatible mechanism and to reduce moral hazard of insured banks, since depositors bearing a deductible will impose greater market
discipline via bank selection and monitoring of risk-taking behavior. In
addition, the deductible also is used to reduce premiums.

We have known that the main factor affecting deposit insurance pre-
mium is the credit risk faced by banks. Here, we use default probability $Q$
to express the credit risk faced by banks. We will now discuss the determi-
nation of $Q$. Assume that the bank is risk-neutral. Based on the theory of
credit risk (Ong, 1999), the expected default probability of banks can be
expressed as follows:

$$Q = \Pr(V \leq B) = \Pr(\ln V_0 + (r - 0.5\sigma_A^2)T + \sigma_A \sqrt{T} Z_T \leq \ln B)$$  \(7\)

$$Pr\left(Z_T \leq \frac{-\ln \left(\frac{V_0}{B}\right) + (r - 0.5\sigma_A^2)T + \sigma_A \sqrt{T} Z_T}{\sigma_A T}\right) = N(-x_2)$$

$$x_1 = \frac{\ln \left(\frac{V_0}{B}\right) + (r_s + \frac{1}{2}\sigma_A^2)T}{\sigma_A \sqrt{T}}, \quad x_2 = x_1 - \sigma_A \sqrt{T}.$$  \(8\)

where $N(\bullet)$ is the cumulative distribution function of the standard normal
distribution, $r_s$ is the risk-free interest rate, $\sigma_A$ is the standard deviation of
the assets of the insured bank, $V_0$ is the value of initial asset of the insured
bank, and $B$ is the combined value of the deposit and interest paid to
creditors by banks during the effective insurance period $T$. Assume that
the capital ratio is $c$ and the value of the deposit is 1, then

$$B = e^{r_s T}$$  \(9\)

Let $V_0 = 1(1 + c)$ and combine it with the above. We then obtain:

$$Q = \Pr(V \leq B) = N(-x_2)$$

$$x_1 = \frac{\ln \left(\frac{1 + c}{e^{r_s T}}\right) + (r_s + \frac{1}{2}\sigma_A^2)T}{\sigma_A \sqrt{T}}, \quad x_2 = x_1 - \sigma_A \sqrt{T}.$$  \(10\)
We take partial derivatives of $Q$ in (10) with respect to $c$ and $\sigma_A$ and obtain

$$
\frac{\partial Q}{\partial c} = \frac{e^{-\frac{1}{2}x_2^2}}{\sqrt{2\pi T}\sigma_A(1+c)} < 0 ,
$$

(11)

$$
\frac{\partial Q}{\partial \sigma_A} = \frac{e^{-\frac{1}{2}x_2^2}}{\sqrt{2\pi}} \left( \frac{\ln \left( \frac{1+c}{e^{rT}} \right) + rST}{\sigma_A^2 T} \right) > 0 .
$$

(12)

If we assume that the risk-free interest rate is equal to the interest rate on deposits, then

$$
\frac{\partial Q}{\partial r_s} = \frac{\partial Q}{\partial r_B} = 0 .
$$

(13)

From inequalities (11) and (12), we see that increasing capital adequacy ratio will decrease the default probability of the insured bank, increasing the standard deviation of bank's assets will increase the default probability of the insured bank and changing the risk-free interest rate will not change the default rate when risk-free interest rate is equal to the deposit interest rate.

Since the factors affecting the premium rate of the deposit insurance not only include the behavior of the deposit insurer (e.g., deductible, upper limit, and investment), but also include the behavior of insured banks (e.g., default rate, capital level, and volatility of assets), the behavior of the banks and the behavior of the deposit insurer are affected by each other. For example, the deductible borne by the depositor will help to reduce the moral hazard of the deposit insurer. Therefore, we discuss and model the behavior of these two parties simultaneously.

For the premium determination, we use the equivalence principle (see Dickson et al., 2009) whereby the expected present value of premiums equals the expected present value of all policy benefits. Consider:

$$
\int_0^T \int_0^T \int_0^T \int_0^T g e^{0 - \int r(u)du} e^{0 - \int r(u)du} dt = \int_0^T \int_0^T (1 - Q) ds Q(1 - k(1 + c) - d) e^{rBt} e^{0 - \int r(u)du} dt
$$

(14)

where $k$ is recovering rate, $g$ is the premium rate of deposit insurance (please note that the premium rate is assumed to have no loading for
Pricing of Deposit Insurance,
and the return rate of investment portfolio \( r \) satisfies
with \( dr(t) = \alpha_1 dr_1 + \alpha_2 dr_2 + \alpha_3 dr_3 \).

Equation (14) can be further written as:

\[
g = \frac{T}{0} \int (1 - Q)Q(1 - k(1 + c) - d)te^{\int_0^t -rB(u)du}E^{\int_0^t e^{-\int_0^t r(u)du}dt}
\]

where

\[
E^{\int_0^t e^{-\int_0^t r(u)du}dt} = e^{A(t) + \frac{1}{2}B(t)}
\]

\[
A(t) = \frac{r_0 - \beta}{\mu_r} (1 - e^{-\beta t}) - \mu_r t
\]

\[
B(t) = \frac{\sigma_r^2}{2\beta^3} (2\beta t - 3 + 4e^{-\beta t} - e^{-2\beta t})
\]

(Mamon, 2004). We use numerical integration to obtain the solution of \( g \).

We now discuss these features from the perspective of both parties. We consider partial derivatives of \( g \), as specified in the equation (15), with respect to the default probability \( Q \), the deductible rate \( d \), capital adequacy rate \( c \), the standard deviation of bank’s assets \( \sigma_{At} \) and the risk-free interest rate \( r_s \). We obtain:
\[
\frac{\partial g}{\partial Q} = \frac{T}{T} - \int_0^T r(u) du = 0 > 0, \text{ when } Q < 0.5 \quad (16)
\]

\[
\frac{\partial g}{\partial d} = -\frac{T}{\int_0^T E e^0 dt} < 0, \quad (17)
\]

\[
\frac{\partial g}{\partial c} = \left( -kQ(1-Q) - \frac{1/2}{\sqrt{2\pi T} T} \right) < 0 \quad (18)
\]

\[
\frac{\partial g}{\partial \sigma_A} = \frac{\partial g}{\partial Q} \frac{\partial g}{\partial \sigma_A} > 0, \text{ when } Q < 0.5 \quad (19)
\]
From inequality (16), we can see that the premium rate is an increasing function of the default probability $Q$ of the insured bank. From inequality (17), we conclude that the premium rate $g$ is a decreasing function of deductible rate $d$. Inequality (18) illustrates that $g$ is also a decreasing function of capital adequacy rate $c$. We also find from equation (20) that the premium rate will increase as the volatility of the bank’s assets increases.

The Pricing Model with Policy Limit

Bank solvency is the main objective of bank regulation, and bank solvency is a primary objective of internal controls at insured banks. Rather than providing all banks with a fixed policy limit of deposit insurance, we examine a flexible deposit insurance policy limit that is inversely related to a bank’s probability of insolvency in determining the deposit insurance premium rate. The purpose for considering the flexible policy limit is similar to that for deductibles—that is, to create an incentive-compatible mechanism to reduce the risk-taking behavior of banks by increasing depositor sensitivity to bank riskiness. Banks having higher limits of deposit insurance because of lower probabilities of insolvency may be positioned to attract more deposits and therefore improve their financial strength. Such signalling would provide strong incentives to banks to improve and maintain their financial strength. Such a mechanism also would signal to customers that they should consider avoiding banks having higher risk.

Let the policy limit be an exponential function of the insolvency probability, i.e., $Ae^{-bQ}$, where $A$ and $b$ are constants. When $b = 0$, the upper limited claim payment is a fixed value. Based on equivalance principle, the premium rate can be written as follows (for the deduction, see Appendix 1):

\[
\frac{\partial g}{\partial r_s} = \frac{\partial g}{\partial r_B} = \frac{\partial g}{\partial Q} \frac{\partial Q}{\partial r_s} + \frac{\int_0^T (1 - 2Q)(1 - k(1 + c) - d) e^{r_B t} e^0 - \int_0^T r(u)du}{\int_0^T e^0 dt},
\]

\[
\text{when } Q < 0.5.
\]
\[
\begin{align*}
    g &= \left\{ \begin{array}{ll}
    T \\
    \int_0^T Q(1 - Q)(1 - k(1 + c) - d)te^{rB^*_t}E \begin{pmatrix} 
        -\int_r^t du \\
        e^0
    \end{pmatrix} \, dt, \\
    - \int_0^T E \begin{pmatrix} 
        -\int_r^t du \\
        e^0
    \end{pmatrix} \, dt
    \end{array} \right. \\
    \text{when } \int_0^T Q(1 - Q)(1 - k(1 + c) - d)te^{rB^*_t}E \begin{pmatrix} 
        -\int_r^t du \\
        e^0
    \end{pmatrix} \, dt &< T \\
\end{align*}
\]

(21)

otherwise.

If the probability of
We use elasticity to reflect the sensitivity of one parameter to changes in another parameter. The elasticity is defined as the measure of a variable’s
sensitivity to a change in another variable. In our cases, elasticity refers
the degree to which the premium responds to changes in the special
parameters.

Taking the partial derivatives of $g$ in (22) with respect to $Q, d, c, r_s(r_B)$,
and $\sigma_A$ we can obtain equations of one order partial derivatives of $g$
with respect to the parameters of $Q, c, d, \sigma_A, r_s$ and further we can obtain
the coefficient of elasticity (see Appendix 2).

**SENSITIVITY ANALYSIS**

It is important to know how sensitive the premium rate $g$ is to changes
in risk parameters such as $Q, c, d$, and $\sigma_A$. We use the elasticity of premium
rate with respect to those parameters to express the sensitivity of the
premium rate. Table 1 lists the values of partial derivative of premium rate
to risk parameters of $Q, c, d$, and $\sigma_A$ without considering the policy limit
when the interest rate is $r_s = 0.05$, and the capital adequacy ratio takes the
values of 0.05 and 0.1, respectively.

From Table 1, we find that the premium rate is most sensitive to the
capital ratio when the capital ratio takes smaller values (e.g., $c = 0.05$), and
thus increasing the capital ratio can effectively decrease the default risk of
banks. However, the standard deviation of bank assets is the most influen-
tial factor for the premium rate when asset volatility takes larger values
(e.g., when $c = 0.10$), and the least influential factor is the risk-free interest
rate (deposit interest rate). The sensitivity of the premium rate to the change of all parameters decreases with increases in the capital ratio except to the change of default rate of the insured banks.

Table 2 lists the values of partial derivatives of the premium rate to risk parameters of $Q$, $c$, $d$, and $\sigma_A$ when the upper limit of claim payment is modeled as a decreasing function of $Q$ and a fixed value ($L$ expresses the upper limit value).

Table 2 shows that the values of elasticity coefficients and premium rate are much smaller compared with those listed in Table 1 for all cases. And also the sensitivity of the premium rate to parameters of default rate and capital ratio is smaller when $c = 0.10$ than when $c = 0.05$. Therefore, the premium rate of deposit insurance with an upper limit is quite robust to changes in these two parameters especially when capital ratio takes larger values (0.10) in our case and with a flexible upper limit. From Table 2, we also find that the difference of parameters (premium rate and upper limit) in Table 2 between using an upper limit and a fixed limit decreases with an increase of capital ratio. The reason is that the default ratio is very small when the capital ratio takes larger values. Therefore, the flexible upper limit is more effective when capital ratio is smaller.

Table 3, Table 4, Figure 1, and Figure 2 present the values of $Q$ and $g$ when $c$ and $\sigma_A$ take different values (the values of other parameters are the same as above) with the case of no upper limit of payment as an example.
Table 3. The Values of Q when c and $\sigma_A$ Take On Different Values

<table>
<thead>
<tr>
<th>$\sigma_A$</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>c = 0.01</td>
<td>0.2431</td>
<td>0.3678</td>
<td>0.4155</td>
<td>0.4413</td>
<td>0.458</td>
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<td>0.4033</td>
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<tr>
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<td>0.085</td>
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<td>0.253</td>
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<td>0.3373</td>
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<td>0.3859</td>
</tr>
<tr>
<td>0.05</td>
<td>0</td>
<td>0.0436</td>
<td>0.1295</td>
<td>0.2021</td>
<td>0.2563</td>
<td>0.2972</td>
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</tr>
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<td>0.0365</td>
<td>0.0914</td>
<td>0.1462</td>
<td>0.1936</td>
<td>0.2333</td>
<td>0.2664</td>
</tr>
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Table 4. The Values of g when c and $\sigma_A$ Take On Different Values

<table>
<thead>
<tr>
<th>$\sigma_A$</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>c = 0.01</td>
<td>0.2427</td>
<td>0.3067</td>
<td>0.3204</td>
<td>0.3252</td>
<td>0.3274</td>
<td>0.3286</td>
<td>0.3292</td>
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<td>0.15</td>
<td>0.1877</td>
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Table 3, Table 4, Figure 1, and Figure 2 show that increasing the capital ratio will decrease the risk exposure of the deposit insurer because the default probability will decrease and the sensitivity of insolvency probability of the insured bank $Q$ and that of premium rate $g$ increase with the increase of capital adequacy ratio $c$ and the standard deviation for the bank’s asset $\sigma_A$. Changing the capital adequacy ratio will change the sensitivity of the premium rate to the standard deviation of banks’ assets. When $c$ has a small value (e.g., $c = 0.01$), the values of $Q$ and $g$ are large, and a change of $\sigma_A$ has only a small effect on them (see the first line of Table 3.
and Table 4). However, when $c$ takes a larger value, the values of $Q$ and $g$ are smaller and a change of $\sigma_A$ has a large effect on them. Slightly decreasing (increasing) $\sigma_A$ will cause $Q$ and $g$ to decrease (increase) largely. The higher value of the premium rate due to higher risk exposure of deposit insurers will somewhat discourage the risk-taking behavior of the insured banks. The higher value of insolvency probability for the insured banks due to higher asset risk indicates that requiring banks to hold adequate capital is not the only way to decrease default risks of insured banks, and monitoring their asset portfolios may be more important. Higher capital ratio, and at the same time, lower asset volatility, may be optimal to minimize insolvency probability of insured banks. An efficient deposit insurance system should combine capital regulation with other measures. In addition, from equation (15), we know that the deductible is inversely
related to the premium rate. Figure 3 describes the relationship between the default rate, the upper policy limit, and the capital ratio when the volatility of assets of the insured banks takes values of 0.03 and 0.06, respectively. We discuss here only the case in which the upper limit is a decreasing function of the default rate of the insured bank.

From Figure 3, we know that the upper limit of the deposit insurance claim is inversely related to the default rate of the insured bank. This arrangement leads to market discipline by encouraging depositors to monitor bank risk taking and other moral hazard behavior so as to reduce the possibility of bankruptcy.

**DISCUSSION OF DEPOSIT INSURANCE PRICING WITH TIME-VARYING DEFAULT PROBABILITY**

In the above sections, we assume that the bank default probability $Q$ is a constant. Actually, $Q$ is a variable changing with time. In this section, we discuss deposit insurance pricing with time-varying default probability. Crook and Bellotti (2010) discuss several time-varying and dynamic models for default risk in consumer loans.

Assume the time-varying default rate in period $t$ is $Q(t)$ (for the models used to forecast the value of $Q(t)$, please see Crook and Bellotti (2010)). When the equation for forecasting $Q(t)$ is established, we only need to substitute $Q$ in equation (15) to get equation (27) and $Q(t)$ equation (22) into $Q(t)$ to get equation (28), and solve these two integral equations to obtain the values of premium rates of deposit insurance ($g$) with deductibles and with policy limit, respectively.
We establish risk-based pricing models of deposit insurance. In order to mitigate moral hazard and to reduce the default probability of the insured banks, we examine the use of a deductible in the deposit insurance scheme, consider the requirements of capital regulation, and propose an upper limit of claim payment that is inversely related to the insolvency probability of the insured bank. We also consider the effects of investment activities of the deposit insurer on the premium rate and select the optimal investment portfolio of deposit insurer assets based on the safety-first criterion. Our results indicate that having an adequate capital ratio is not the only way to prevent bankruptcy, and the volatility of asset portfolio may be more important for regulators to monitor. We solve for the optimal premium in the context of a modified deposit insurance policy having a deductible and policy limit, consistent with the regulatory goal of preventing insolvency.

REFERENCES


APPENDIX 1

Based again on the equivalence principle, we conclude that the premium rate must satisfy the equation:

\[
T \int_0^t r(u) du \geq 0 \int_0^t (1 - Q) ds (1 - k(1 + c) - d) e^{r_B t} e^0 dt = \min \left\{ T \int_0^t (1 - Q) ds (1 - k(1 + c) - d) e^{r_B t} e^0 dt, \right. \\
\left. \int_0^t Q (1 - Q) (1 - k(1 + c) - d) e^{r_B t} e^0 dt \right\}
\]  

(18)

Since

\[
T \int_0^t (1 - Q) ds (1 - k(1 + c) - d) e^{r_B t} E e^0 dt, =
\]

and

\[
T \int_0^t (1 - Q) ds (A e^{-bQ} - k(1 + c)) E e^0 dt =
\]
rewriting equation (18), we get

\[
\begin{align*}
\int_{0}^{T} \left\{ -\int_{0}^{t} r(u) du \right\} dt = \\
\int_{0}^{T} \left\{ -\int_{0}^{t} r(u) du \right\} dt,
\end{align*}
\]

when

\[
\begin{align*}
\int_{0}^{T} \left\{ -\int_{0}^{t} r(u) du \right\} dt < \\
\int_{0}^{T} \left\{ -\int_{0}^{t} r(u) du \right\} dt,
\end{align*}
\]

and

\[
\begin{align*}
\int_{0}^{T} \left\{ -\int_{0}^{t} r(u) du \right\} dt = \\
\int_{0}^{T} \left\{ -\int_{0}^{t} r(u) du \right\} dt,
\end{align*}
\]

otherwise. Based on equation (19), the premium rate can be written as:
\[
g = \begin{cases} 
  T 
  \int_0^T Q(1 - Q)(1 - k(1 + c) - d)te^{r_b t} \left\{ \begin{array}{c} 
  - \int_r^t du \\
  e^0 
  \end{array} \right\} dt, \\
  \text{when } \int_0^T Q(1 - Q)(1 - k(1 + c) - d)te^{r_b t} \left\{ \begin{array}{c} 
  - \int_r^t du \\
  e^0 
  \end{array} \right\} dt < \\
  T 
  \int_0^T Q(1 - Q)(Ae^{-bQ} - k(1 + c)e^{r_b t})tE \left\{ \begin{array}{c} 
  - \int_r^t du \\
  e^0 
  \end{array} \right\} dt, \\
  \text{otherwise.}
\end{cases}
\]
APPENDIX 2

Following are formulas for calculating the elasticity coefficient of premium rate $g$ with respect to parameters $Q, c, d, \sigma_A, rs(r_B)$: (see next page for equation 22)
\[ \eta_{gQ} = \frac{\partial g \cdot Q}{\partial Q} = \begin{cases} 
\int_0^T Q(1 - 2Q)(1 - k(1 + c) - d)te^rE \left( \int_0^t r(u)du \right) dt \nT \int_0^T e^0 \left( \int_0^t r(u)du \right) dt 
\end{cases} \]

\[ \text{when} \int_0^T Q(1 - Q)(1 - k(1 + c) - d)te^rB^tE \left( \int_0^t r(u)du \right) dt < \]

\[ T \int_0^T \left( A e^{-bQ} - k(1 + c)e^{rB^t} \right) tE e^0 \left( \int_0^t r(u)du \right) dt \]

\[ \int_0^T \left( 1 - 2Q - bQ(1 - Q) \right) \left( A e^{-bQ} - (1 - 2Q)k(1 + c)e^{rB^t} \right) tE e^0 \left( \int_0^t r(u)du \right) dt \]

\[ \begin{cases} 
\int_0^T e^0 \left( \int_0^t r(u)du \right) dt 
\end{cases} \]

otherwise.
\[ \eta_{gc} = \frac{\partial g \cdot c}{\partial c \cdot g} = \frac{c}{g} \frac{\partial g}{\partial c} \frac{\partial Q}{\partial g} - \frac{1}{g} \left( \int_0^T Q(1 - Q) k t E e^{-r B t} dt \right) \]

which is equal to \(0\) when

\[ \int_0^T Q(1 - Q) (1 - k(1 + c) - d) e^{-r B t} dt \geq 0 \]

and

\[ \eta_{g\sigma_A} = \frac{\partial g \cdot \sigma_A}{\partial \sigma_A \cdot g} = \frac{\partial g}{\partial Q} \frac{\partial Q \cdot \sigma_A}{\partial \sigma_A \cdot g} \]

and

\[ \eta_{gr_s} = \frac{\partial g \cdot r_s}{\partial r_s \cdot g} = \frac{r_s}{g} \left( \frac{\partial g}{\partial r_s} \frac{\partial Q}{\partial r_s} - \frac{1}{g} \left( \int_0^T Q(1 - Q) k(1 + c) e^{r B t} t^2 E e^{-r B t} dt \right) \right) \]