Modeling Competition in a Market for Natural Catastrophe Insurance\textsuperscript{1}

Yang Gao, Linda Nozick, Jamie Kruse, and Rachel Davidson\textsuperscript{2}

\textbf{Abstract:} We model the structure of the primary natural catastrophe insurance market using a static perfect information Cournot-Nash noncooperative game while integrating a state-of-the-art regional catastrophe loss estimation model. This approach, which generates an optimal operational cost surface, contributes to the creation of an internal risk model to measure the adequacy of a firm’s capital and financial risk management. We apply the modeling framework to a full-scale case study for hurricane risk (flood and wind combined) for residential buildings in eastern North Carolina. The results from our study indicate that the level of concentration in the primary insurance market can lead to significant differences in the firm’s operational decisions (e.g., choice in reinsurance and retained or capped surplus). As expected, a more competitive primary insurance market reduces the profitability of primary insurers, but is attractive to homeowners; hence there is an important balance to be maintained between covering as many properties as possible in the region while maintaining the profitability/solvency of the carriers. Further, our results suggest that encouraging catastrophe reserves for insurance companies can reduce their likelihood of insolvency. [Key words: catastrophe insurance, natural hazard, Cournot-Nash, internal risk management.]

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\textsuperscript{2}Yang Gao, Civil and Environmental Engineering Department, Cornell University, Ithaca, NY 14850 USA, e-mail: yg245@cornell.edu.
Linda Nozick, Civil and Environmental Engineering Department, Cornell University, Ithaca, NY 14850 USA, e-mail: lkn3@cornell.edu.
Jamie Kruse, Department of Economics, East Carolina University, Greenville, NC 27858 USA, e-mail: krusej@ecu.edu.
Rachel Davidson, Civil and Environmental Engineering Department, University of Delaware, Newark, DE 19716 USA, e-mail: rdavidso@udel.edu.
CATASTROPHE INSURANCE MARKET

Substantial growth in coastal populations has led to a dramatic increase in the consequences of natural disasters (Kunreuther, 1998). Natural disaster catastrophe loss insurance is one mechanism to address this growing risk. Unfortunately, this market faces challenges, leaving many properties either uninsured or substantially underinsured. Many have proposed possible explanations for challenges to the catastrophe risk insurance market, related to both the nature of catastrophe risk itself and the structure of the market. In particular, catastrophe losses tend to be highly correlated in space and characterized by “fat tail” distributions, making it especially difficult for an insurer to avoid the possibility of insolvency (e.g., Kousky and Cooke, 2012). The difficulty in precisely estimating catastrophe risk, limited insurer capacity to cover potentially large losses, behavioral biases manifested in insurance purchase decisions, moral hazard, charity hazard, and tax and agency costs of holding capital all have been noted as possible contributors to the difficulties in establishing a healthy catastrophe insurance market (e.g., Jaffee and Russell, 1997; Froot, 2001; Kunreuther et al., 2002; Kunreuther, 2006; Kousky, 2011; Paudel, 2012; Medders, Nyce, and Karl, in press). Also, Grace et al. (1998), Grace et al. (2003), Kleindorfer and Klein (2003), and Kunreuther and Michel-Kerjan (2009) identify a passive supply-demand relationship and likely inefficient and misguided government intervention as conditions that have exacerbated the stress on the market.

With respect to demand, property owners often do not fully insure their property nor do they invest in pre-event mitigation activities that can reduce disaster losses (Kunreuther, 1998; Kreisel and Landry, 2004; Kunreuther and Pauly, 2004; Dixon et al., 2006). Property owners are likely to have insufficient financial resources to recover losses and may require government relief (Kunreuther and Pauly, 2004).

On the supply side, insurers receive a stable flow of premiums to address a liability stream that is highly variable. This mismatch between stable receipts and sporadic but very large expenditures must be addressed through intertemporal risk spreading, which is difficult to achieve (Jaffee and Russell, 1997). For example, some data from the reinsurance industry implies that insurance companies tend to retain, rather than share, their large-event risk. In fact, the vast majority of primitive catastrophe risk in the economy is being retained (Froot, 2001). Due to the difficulty in managing risk, insurers have limited renewals of existing contracts, and issuing of new contracts is inadequate in at-risk regions (U.S. GAO, 2007). In the worst case, disaster events can cause insolvency, as happened, for example, to eight small Florida insurance companies after Hurricane Andrew (Grier,
1996). Despite the challenges for insurers to survive, state legislators and insurance regulators appear to have suppressed insurance prices, compressed rate differences between high and low risk areas, and imposed significant cross-subsidies from low-risk to high-risk areas (Kleindorfer and Klein, 2003).

Various private market and public policy interventions have been proposed to address the challenges. Cole et al., (2011) discuss alternate forms of post-loss financing. Other policy interventions include securitizing catastrophe risk with catastrophe bonds and derivatives, allowing insurers to establish tax-deferred catastrophe reserves, providing state and federal government reinsurance programs, allowing insurance premiums to accurately reflect risk to encourage mitigation, introducing long-term property insurance tied to a mortgage, establishing an all-perils insurance policy, requiring all building owners to purchase insurance, offering insurance vouchers to low-income residents in disaster prone areas, and establishing an independent federal office to operate a catastrophic reinsurance program (Boyer and Nyce, 2013; Harrington and Niehaus, 2001; Kunreuther and Michel-Kerjan, 2009; Kunreuther and Pauly, 2006; Litan, 2006; Niehaus, 2002; Paudel, 2012; U.S. GAO, 2007).

Given the complexity of insurance markets with the potential for catastrophic loss, there is a clear need for comprehensive examination of the interactions between key stakeholders (property owners, insurers, reinsurers, and government) so that effective methods to restructure the catastrophe loss insurance market can be identified. In this study, we use a game theoretic modeling framework to capture the strategic relationship of stakeholders in the market. Within this framework, we examine performance of a voluntary catastrophe insurance market as the level of competition within that market changes. More specifically, we model the structure of the primary insurance market using a static perfect information Cournot-Nash noncooperative game while integrating (1) a utility-based homeowner decision model representing demand; (2) a stochastic optimization model to optimize reinsurance decisions by the primary insurer(s); and (3) a state-of-the-art regional catastrophe loss estimation model. We allow the number of primary insurers to increase from one (monopoly) to many within the Cournot-Nash framework. In addition, we analyze a cooperative solution for each level of market concentration by symmetric insurers. Although we use a Cournot-Nash game to model the interaction between insurers, by also characterizing the cooperative (joint profit maximizing) solution we can benchmark the range of outcomes viable under a repeated game with trigger strategies for all possible discount rates.

This modeling framework is applied to a full-scale case study for hurricane risk (flood and wind combined) for residential buildings in
eastern North Carolina. We assume heterogeneous homeowners and insurers with identical cost structures, and focus on the equilibrium supply-demand relationship under conditions that range from monopoly and oligopoly to market structures that emulate near-perfect competition. Furthermore, we examine impacts of market concentration on the profitability and insolvency of a single insurer, as well as its decisions to hedge risk as the number of firms increases.

Following a summary of relevant literature in Section 2, the formulation of the single-shot Cournot-Nash model is described in Section 3. We derive inverse demand functions for insurance using an agent-based homeowner decision model and an insurer cost function, both founded on a state-of-the-art loss model. In Section 4, we present a modeling framework that includes interacting models of loss, homeowner decisions, and insurer decisions. Section 5 presents inputs required for the case study application for hurricane risk to residential buildings in eastern North Carolina. The case study results are presented in Section 6, including discussion of how equilibrium price, insurance penetration, insurer’s performance, and reinsurance decisions interact and change as the number of insurers in the market increases. Finally, the summary of this research as well opportunities for future work are presented in Section 7.

**REVIEW OF INSURANCE MARKET MODELS**

The theoretical insurance literature has focused primarily on a monopoly insurer, monopolistically competitive market structure (where insurers compete with heterogeneous but highly substitutable insurance products), or perfect competition (e.g., Boyer and Nyce, in press; Joskow, 1973; Rothschild and Stiglitz, 1976; Stiglitz, 1977; Spence, 1978). Although consistent empirical findings indicate an oligopolistic market structure (Nissan and Caveny, 2001; Murat et al., 2002), there are few models of oligopolistic insurance markets for which the insurers make positive profits. Sonnenholzner and Wambach (2004) give an excellent overview of these models. Among the papers on oligopolistic insurance markets, Schlesinger and Schulenburg (1991) analyze a model with heterogeneous insurance products and Polborn (1998) applies the Bertrand game to model oligopolistic interactions between risk-averse insurers.

Cournot-Nash and Bertrand models of oligopoly markets are examples of classical static non-cooperative games (Tirole, 1988; Lynne and Dan, 2008). The Bertrand framework has been used to represent oligopoly insurance markets—where players/insurers compete based on price in Polborn (1998) and Sonnenholzner and Wambach (2004). Most of the
oligopoly insurance models consider product differentiation and/or imperfect information (e.g., Rothschild and Stiglitz, 1976; Stiglitz, 1977; Spence, 1978; Schlesinger and Schulenburg, 1991; Polborn, 1998; Wambach, 2000, Boyer and Nyce, 2013). In contrast, under Cournot-Nash competition, equilibrium prices can remain substantially above marginal cost so long as the number of players is not large. Finally, there are substantial capital requirements associated with the quantity of insurance that a firm offers; hence in the short run it is difficult to change volume rapidly. The evidence on economies of scale in the insurance industry demonstrates increasing, constant, and decreasing returns to scale for firms below, at, or above median size, respectively, with a significant number of firms in each decile operating at constant returns to scale (Cummins and Xie, 2013).

The static Cournot-Nash model is a one-shot game. The reality, of course, is that insurers/players interact repeatedly within the market. Although we have not opted for a dynamic framework, we are able to characterize the range of possible outcomes by identifying the Cournot-Nash solution and the joint profit maximizing solution for each number of players as benchmarks.

Finally, we choose not to address either moral hazard or adverse selection. Russell and Jaffee (1997) argue that since the event creating the risk is beyond the control of those who are insured, moral hazard and adverse selection tend not to be as critical an issue for catastrophe insurance.

**COURNOT-NASH MODEL OF CATASTROPHE INSURANCE MARKETS**

Our base case of oligopolistic behavior in the primary catastrophe insurance market is a single-shot noncooperative game. All insurers in the market compete once and make their decisions simultaneously. We assume that there are no disparities in information and all insurers are homogeneous. In other words, each insurer faces the same cost structure and is equally capable of handling all levels of demand. For simplicity, we assume that insurers only offer full coverage insurance. Each property owner, at most, purchases one insurance contract from a single insurer.

More specifically, consider $n$ homogeneous private insurers selling catastrophe insurance for residential buildings in a region. We assume the region is divided into smaller risk regions $v \in V$ defined to allow homeowner risk attitudes and insurer premiums to vary geographically. Insurer $j$’s decisions in the Cournot-Nash game—the total coverage offered in each risk region $v$ is denoted as $q_{vj}$ in terms of annual expected insured loss.
(from natural disasters). The price (per unit) of insurance policies sold in a risk region is assumed to have a relationship with the total volume offered by all insurers in this region. More specifically, assume that there is a distinct demand function for each risk region: \( Q_v = D_v(p_v), \forall v \in V \)

with its inverse: \( p_v = P_v(Q_v) = D_v^{-1}(Q_v), \forall v \in V \), where \( Q_v \) refers to total losses insured by all insurers in the entire region and \( p_v \) refers to price per dollar coverage. Since the total insured losses actually come from all the insurers in the market, a single insurer \( j \)'s price-demand relationship also depends on its rivals’ actions, and can be presented as:

\[
p_v = P_v(q_{vj}, \sum_{k \neq j} q_{vk}) = P_v(q_{vj}, Q_{-vj}).
\]

The cost function for insurer \( j \) to operate the business in all risk regions is denoted as \( C_j(\sum_{v \in V} q_{vj}) \). Finally, insurer \( j \)'s annual net profit can be described as the premiums paid to insurer \( j \) by those they insure minus the cost of providing the insurance over a year (Eq. 1). Note that the first term is a function of the coverage provided by all insurers since that determines the price, while the second term is a function of only the coverage offered by the insurer \( j \). Expressing the individual insurer’s objective as a function of its decisions/strategies and its competitors’ facilitates deduction of the equilibrium conditions.

\[
\pi_j(\sum_{v \in V} q_{vj}, \sum_{v \in V} Q_{-vj}) = \sum_{v \in V} q_{vj} P_v(q_{vj}, Q_{-vj}) - C_j(\sum_{v \in V} q_{vj}) \quad q_{vj} \geq 0
\]

We assume each insurer is a net profit maximizer and formulates a best-response function that recognizes the strategic interaction with other insurers serving the same locations. If the problem is differentiable, the optimal solution for net profit maximization should satisfy the first-order conditions (Eq. 2), based on the envelope theorem (Mas-Colell et al., 1995), which also refers to the relationship that marginal cost equals to marginal revenue:

\[
\frac{\partial \pi_j}{\partial q_{vj}} = P_v(q_{vj}, Q_{-vj}) + q_{vj} \frac{\partial P_v(q_{vj}, Q_{-vj})}{\partial q_{vj}} - \frac{\partial C_j(\sum_{v \in V} q_{vj})}{\partial q_{vj}} = 0 \quad \forall v \in V
\]

Denote insurer \( j \)'s actions that satisfy the first-order conditions as a vector of reaction functions (i.e., coverage provided in each region \( v \) or best responses): \( q_j = (q_{vj}^*, \forall v \in V) \). Since the insurers are all homogeneous, they will have the same reaction functions. Thus, by symmetry, we would have insurer \( j \)'s rivals’ actions as: \( Q_{-vj} = (n-1)q_{vj}^*, \forall v \in V \). By substituting these relations into the first-order conditions (Eq. 2), they can be rewritten as:

\[
P_v(nq_{vj}^*) + q_{vj}^* \frac{\partial P_v(nq_{vj}^*)}{\partial q_{vj}} - \frac{\partial C_j(\sum_{v \in V} q_{vj})}{\partial q_{vj}} = 0 \quad \forall v \in V
\]
Solving Equations 3 explicitly, the reaction function (best response function of insurer \( j \)), denoted as vector \( q_j = R_j(q_{-j}) \), where \( q_{-j} \) refers to all its opponents’ reactions, has a closed form as:

\[
(q_{-j})_j(q_j^*) = \left[ \frac{\partial C_j(\Sigma_{v \in V} q_{vj})}{\partial q_{vj}} / (\Sigma_{v \in V} q_{vj}) \right] - P_v(nq^*_v) / (\partial q_{vj})
\]  

**HOMEOWNER AND INSURER DECISION MODELS TO COMPUTE INVERSE DEMAND AND COST FUNCTIONS**

Application of the Cournot-Nash equilibrium model in Section 3 requires defining inverse demand functions \( p_v \), \( \forall v \in V \) and a cost function \( C_j(\Sigma_{v \in V} q_{vj}) \), for simplicity, denoted as \( C_j(\Sigma_{v \in V} q_{vj}) \) or \( C_j \). A key contribution of this work is to demonstrate how those functions can be developed using a modeling framework that includes interacting models of loss, homeowner decisions, and insurer decisions. This section describes the framework, and then how it is used to define the functions needed for the Cournot-Nash model.

**Overview of Interacting Loss and Decision Models**

The framework is illustrated for each primary insurer. It includes three models and represents three main players (Fig. 1). The loss model is a simulation that combines hazard, inventory, and damage modules to compute a probability distribution of losses for each group of buildings (defined by location and type) and each possible hurricane in the study area. The inputs required are similar to those that can be obtained from any
regional loss estimation model, such as HAZUS-MH 2.1 (FEMA, 2012) or the Florida Public Hurricane Loss Model (FPHLM, 2005). The primary insurer determines what premiums to charge for policies at a specified deductible, and what reinsurance to purchase, and each homeowner (defined by his home’s location and type) responds by deciding whether or not to purchase that insurance. Specifically, the primary insurer model is a stochastic optimization in which the objective is to maximize net profit. The homeowners’ decision-making is modeled as a utility maximization problem. The homeowner and loss models together are used to develop the inverse demand functions. The insurer, homeowner, reinsurance, and loss models are used to develop the cost function.

Definitions

Building inventory. The inventory of residential buildings is divided into groups, where each is defined by its geographic area unit or location \( i \) (e.g., census tract), building category \( m \), resistance level \( c \), and risk region \( v \). Building categories \( m \) are defined based on architectural features and are assumed to perform similarly in hurricanes and have similar value (e.g., one-story home with a garage and hip roof). A building’s resistance level \( c \) represents its vulnerability and is a function of structural details that define the probability of damage given wind speed and flood depth. As defined previously, risk regions \( v \) are larger geographic areas comprised of many area units \( i \). They are defined to allow homeowner risk attitudes and insurer premiums to vary by location, but at greater aggregation than area units. The building inventory, which we assume to be constant over time, is defined using \( X_{incr} \) the number of buildings of type \( i, m, c, v \).

Stakeholders. The collection of homeowners in the study area are disaggregated by location \( i \), building category \( m \), building resistance level \( c \), and risk region \( v \). Since homeowners differ based on \( m, c, v \), and possibly their risk attitude, heterogeneous behavior of homeowners is considered via a utility function that includes a probabilistic representation of risk attitude and arguments that capture the vulnerability of each homeowner’s property. We assume all insurers are homogeneous net profit maximizers and may choose to purchase one layer of catastrophe risk excess of loss reinsurance to manage/transfer part of their liabilities. Capital markets are not considered for these primary insurers. The reinsurance market is assumed to accommodate demand by each primary insurer at a standard price level.

Time. The durations of the time steps \( t \) vary (a few days to a few weeks). They are defined to be short enough so that we can reasonably assume no two hurricanes occur in the same time period, and so that the probability a hurricane occurs in one time period is equal across time periods. Since
hurricane occurrence varies during the year, time periods are shorter, for example, in September when hurricanes are more likely than in June when they are less likely. Since hurricanes are virtually nonexistent from mid-December to mid-May, we omit those months from the year.

Hazard. Only insurance policies for both hurricane-related wind and storm surge flooding are considered. The hurricane hazard is represented by an efficient set of probabilistic hurricane scenarios \( h \in (1, \ldots, H) \), defined as tracks with along-track parameters that determine the intensity, including central pressure deficit and radius to maximum winds. Each hurricane scenario has an associated hazard-adjusted annual occurrence probability \( P_h \) such that when probabilistically combined, the set of hurricane scenarios represents the regional hazard (Apivatanagul et al., 2011). For each hurricane, wind speeds and surge depths are estimated throughout the study area; in a sense, each hurricane scenario represents all hurricanes that would produce similar wind speeds and surge depths in the study area. The occurrence probability for period \( t \) is calculated from the annual occurrence probability using the historical relative frequency of events over the course of the hurricane season (Peng, 2013).

A series of hurricanes in quick succession can generate very different outcomes for an insurer than the same number of hurricanes evenly spread over time. We therefore define a long-term (thirty-year) timeline of hurricanes for which each is referred to as a scenario \( s \in (1, \ldots, S) \). (To avoid confusion, we refer to a single hurricane event scenario as simply a hurricane \( h \).) Each scenario \( s \) is a \( 1 \times T \) vector, where \( T \) is total number of time periods, and for each time period \( t \), either one of the possible hurricanes \( h \) occurs, or no hurricane occurs. For ease of notation, we refer to the case of no hurricane as \( h = H + 1 \). Each scenario has an occurrence probability \( P_s \), such that \( \Sigma_s P_s = 1 \). The complete set of scenarios (on the order of hundreds or perhaps thousands) is defined so that it has the same key characteristics as the full set of \( (H + 1)^T \) scenarios that is theoretically possible. See Peng (2013) for more details on the creation of the suite of scenarios.

**Deriving Inverse Demand Functions**

The demand for catastrophe insurance is assumed to be separable by risk region \( v \), so there exist \( V \) independent inverse demand curves, i.e., price-demand relationships \( p_v = P_v(Q_v), \ \forall v \in V \). To develop that curve for one region \( v \), we solve the homeowner decision model for every homeowner in the region at a specified price level \( p_v \) (i.e., price per dollar coverage in risk region \( v \)) to compute the demand \( Q_v \) (i.e., total expected coverage in region \( v \)) at that price level. The calculation is iterated at many values of \( p_v \) to obtain the pairs \( (p_v, Q_v) \), data which we then use to fit an inverse demand curve.
The homeowner decision model is defined on an annual basis and for a single homeowner in location \( i \), building category \( m \), resistance level \( c \), and risk region \( v \). It can be solved separately for buildings of each type \( i, m, c, v \), and thus the computation can be parallelized. For a homeowner of type \( i, m, c, v \), the model determines whether or not to buy insurance at the specified price \( p_v \) given a specified deductible \( d \). Specifically, it yields the binary decision variable \( w_{imcv} \), which is one if insurance is purchased, and zero otherwise.

We denote the loss in the event of hurricane \( h \), for a single home of type \( i, m, c, v \), as \( L_{imcv}^h \), which is estimated from the loss model. The annual premium, denoted as \( Z_{imcv} \), is defined as the price per dollar coverage times the annual expected loss (Eq. 6), and the amount the homeowner actually pays as a deductible in the event of hurricane \( h \) is the minimum of the loss experienced and the deductible (Eq. 7).

\[
Z_{imcv} = p_v \Sigma_h p^h_{imcv} \quad \forall i, m, c, v \quad (6)
\]

\[
B_{imcv}^h = \min\{L_{imcv}^h, d\} \quad \forall i, m, c, v \quad (7)
\]

We assume each homeowner has a maximum budget for homeowner insurance equal to a specified percentage \( \kappa_v \) of his home value \( V_m \) (Eq. 8); and insurers will only offer insurance if the premium is greater than some specified value \( \rho \) (Eq. 9). Finally, the decision variables \( w_{imcv} \) must be zero or one.

\[
Z_{imcv} \leq \kappa_v V_m \quad \forall i, m, c, v \quad (8)
\]

\[
Z_{imcv} \geq \rho \quad \forall i, m, c, v \quad (9)
\]

\[
w_{imcv} = \{0, 1\} \quad \forall i, m, c, v \quad (10)
\]

We assume the decision is made by maximizing utility, with risk preferences represented by the utility function \( U(x) = 1 - e^{-\theta_{imcv} x} \), where \( \theta_{imcv} \) is the Arrow-Pratt measure of risk aversion for homeowners of type \( i, m, c, v \). For \( \theta_{imcv} > 0 \) homeowners are risk averse, which is necessary for insurance to be a possibility. In the case study, we assume that all homeowners of type \( i, m, c, v \) have the same risk aversion parameter \( \theta_{imcv} \), and that the values of \( \theta_{imcv} \) within a risk region \( v \) are lognormally
distributed with a specified mean \( \log(\theta_v) \) and standard deviation one (Eq. 11).

\[
\log(\theta_{imcv}) \sim N(\log(\theta_v),1) \quad \forall i, m, c, v
\]  

(11)

The homeowner’s objective function (Eq. 12) is to maximize the probability weighted sum of utilities over all possible hurricanes \( h \) if he buys insurance (first term) and if he does not (second term). In the first case, the homeowner pays the premium and loss up to the deductible. In the second case, the homeowner pays the loss due to building damage only. Note that when \( h = H + 1 \), no hurricane occurs, and the loss is zero.

\[
\text{Max} \ w_{imcv}[\Sigma_h P^h(U(Z_{imcv} + B^h_{imcv}))] + (1 - w_{imcv})[\Sigma_h P^h(U(L^h_{imcv}))]
\]  

(12)

The homeowner model is defined by optimizing the objective function (12) subject to Constraints (6) to (10). We denote the solution to that model for a homeowner of type \( i, m, c, v \) as \( w^*_{imcv} \). If \( X_{imcv} \) is the number of buildings of \( i, m, c, v \), then the total expected insured losses \( Q_v \) for insurers at price level \( p_v \) and deductible \( d \) is as given in Eq. 13. Computing \( Q_v \) for many different prices \( p_v \), we can then fit an inverse demand function \( p_v = (Q_v) \).

\[
Q_v = \Sigma_{imc} w^*_{imcv} (\Sigma_h P^h L^h_{imcv} X_{imcv}) \quad \forall v
\]  

(13)

A logical extension of this work would be to replace the homeowner formulation with a discrete choice model or other formulation that accounts for some of the documented homeowner biases and heuristics (Kunreuther, 2006).

**Defining Cost Function**

The cost function in the Cournot-Nash model \( C_j(\Sigma_v q_v) \) defines the cost for insurer \( j \) to operate the business in all risk regions \( v \), and is a function of all \( q_v \), the set of demands satisfied by insurer \( j \) in each risk region \( v \) (i.e., total expected insured losses in region \( v \)). We compute this function in three steps. First, given a price per dollar expected loss \( p_v \), we use the insurer optimization model to determine how much risk the insurer will transfer (specifically, the attachment point \( A \) and maximum limit \( M \) of the excess of loss reinsurance treaty) so as to maximize its profit. As described previously, the insurer optimization interacts with the homeowner model, which determines which homeowners will buy insurance \( w^*_{imcv} \), and thus
the homeowner demand \( q_{ij} \) at the specified price level. Second, given the homeowner demand and reinsurance treaty parameters, we can calculate the cost to the insurer \( C_j \). Third, we repeat that calculation for different price levels \( p_v \) and combine it with the homeowner demand to obtain a set of data triplets \( (p_v, q_{ij}, C_j) \). Finally, we fit a cost function to that data that relates \( C_j \) to \( q_{ij} \) for all risk regions and can be used as input in the Cournot-Nash model. In the next subsection, we define insurer cost more precisely, and in the subsequent subsection we present the insurer decision model formulation.

**Definition of Insurer Cost**

We approximate the insurer’s total cost in this model as the sum of the operational costs and insurer’s liabilities. The operational cost is assumed to be a fixed portion \( \tau \) (35\% in the case study on personal communication with John Aquino, WillisRe) of annual expected insured losses (i.e., each dollar of coverage costs $0.35 for the insurer to provide).\(^3\) Insurer liabilities include the coverage of insured homeowner losses reduced by the risk transferred to the reinsurer, plus payments to the reinsurer. For the assumption of a single layer of catastrophe risk excess of loss reinsurance, in the event of a hurricane \( h \), the loss to all insured buildings \( L^h \) is divided among the homeowners, primary insurer, and reinsurer as in Figure 2. The variables \( A \), \( M \), and \( \beta \) are the attachment point, maximum limit and co-participation percentage of the reinsurance treaty, respectively. If hurri-

\(^3\)See Boyer and Nyce (2013) for an alternate formulation to represent the insurer’s marginal cost function. The Boyer and Nyce model assumes that the insurer’s marginal cost of capital is increasing in loss and therefore generates convex cost function. \( \tau \) in our model is intended to capture underwriting costs and would contribute to a constant marginal cost formulation.
cane \( h \) occurs, each homeowner pays the first portion of the loss up to the deductible; the reinsurer pays \( \beta \% \) of any loss above the attachment point \( A \) and up to a maximum limit \( \beta \% \) of \( (M - A) \); and the primary insurer pays the remaining loss. The excess of loss reinsurance policy itself requires that the insurer pay the reinsurer a premium of \( b \), and in the event of a hurricane \( h \), an additional reinstatement premium to reinstate the limit \( M \). Overall, the insured losses not covered by the deductibles and the reinsurance policy, plus the payments to the reinsurer, are considered the insurer’s liability.

Purchasing a reinsurance policy with low attachment point and high limit reduces the insured losses/liabilities incurred by the primary insurer, thus reducing the risk and stabilizing profit over the long term. However, this must be balanced against the result that a high reinsurance premium reduces the primary insurer’s net profit in each time period. Interrelated choices about homeowner insurance pricing that affect homeowner purchase decisions that, in turn, affect the loss distribution ultimately influence the optimal reinsurance strategy for the primary insurer.

**Insurer Decision Optimization Model**

The total loss insured by a primary insurer depends on market share. Since insurers are assumed to be homogeneous in this study, they have the same optimal cost structure under different insured loss levels, and are assumed capable of handling all levels of insured losses.

For a given price level, the insurer determines how much risk to transfer to the reinsurer (i.e., values of \( A \) and \( M \)), by maximizing its expected net profit over the set \( S \) of long-term hurricane scenarios described previously. The total insured loss \( L^h \) (Eq. 14) and the loss borne by the homeowners in the form of deductibles \( B^h \) (Eq. 15) are:

\[
L^h = \sum_{imcv} w^*_{imcv} L^h_{imcv} \cdot X_{imcv} \quad (14)
\]

\[
B^h = \sum_{imcv} w^*_{imcv} B^h_{imcv} \cdot X_{imcv} \quad (15)
\]

**Reinsurance Coverage.** We define \( q^h \) to be the loss above \( A \) and below \( M \) for hurricane \( h \) (Eq. 16). If the loss exceeds the attachment point \( A \), then \( \beta q^h \) is recovered from the reinsurer and the primary insurer incurs \( (1 - \beta) q^h \), where \( \beta \) is a specified input constant (Fig. 2).

\[
q^h = \min \{ \max \{ L^h - A, 0 \}, M - A \} \quad \forall h \quad (16)
\]
For a given scenario \( s \) and time \( t \), which hurricane \( h \) (or no hurricane) happens is known, so we can define \( e_s^{y} \), the loss between attachment \( A \) and limit \( M \) for scenario \( s \) and year \( y \) in Equation 17, where \( q_h^{s t h} \) is a binary indicator variable that is one if hurricane \( h \) happens in scenario \( s \) at time \( t \) and zero otherwise. Since at most one hurricane can happen in a time period \( t \), \( \sum q_h^{s t h} \leq 1 \) \( \forall s, t \). The set \( \omega(y) \) defines the set of time periods \( t \) in year \( y \).

\[
e_s^{y} = \sum_{t \in \omega(y)} \sum_{h} q_h^{s t h} \forall s, y
\]  

(17)

Reinsurance premium. In each year \( y \), the primary insurer pays the reinsurer a base premium \( b \), and in the event of a hurricane \( h \), it also pays a reinstatement premium to reinstate the limit \( M \). The base premium is computed as the expected loss the reinsurer is responsible for multiplied by one plus a loading factor \( \phi \), plus the standard deviation \( \sigma \) of the net reinsurer loss multiplied by \( \beta \) and a user-specified constant \( g \) (Eq. 18) (Kunreuther and Michel-Kerjan, 2009).

\[
b = (1 + \phi)[\sum h \beta q_h^{h}] + g \beta \sigma
\]  

(18)

The loading factor \( \phi \) represents the reinsurer’s share of the loss adjustment expenses, its own expenses, and its profit, and \( g \) represents the reinsurer’s risk aversion. The \( \sigma \) is the standard deviation over all scenarios \( s \) and years \( y \) of the reinsurer’s loss, \( e_s^{y} \), less the reinstatement premium for scenario \( s \) and year \( y \). The reinstatement premium is a pro rata amount of the expected reinsurer loss without adjusting for the length of the treaty’s remaining term. That is, it equals the expected loss multiplied by the percentage of the original coverage that was used \((e_s^{y}/(M-A))\). The total reinsurance premium for scenario \( s \) in year \( y \), therefore, is the sum of the base reinsurance premium and the reinstatement payment:

\[
r_s^{y} = b + \left( \frac{e_s^{y}}{M-A} \right)[\sum h \beta q_h^{h}] \forall s, y
\]  

(19)

Primary insurer’s profit. Equation (20) defines the insurer’s net profit, \( F_s^{y} \), in scenario \( s \) and year \( y \). The terms are, in turn, the total homeowner premiums collected, transaction cost, total loss to insured buildings minus deductibles, actual loss recovered from the reinsurer, and reinsurance premium.

\[
F_s^{y} = \sum v P_v Q_v - \tau(\sum v Q_v) - \sum_{t \in \omega(y)} \sum_{h} q_h^{s t h} L^h + \\
\sum_{t \in \omega(y)} \sum_{h} q_h^{s t h} B^h + \beta e_s^{y} - r_s^{y} \forall s, y
\]  

(20)
Primary insurer’s accumulated surplus. In reality, the funds available to the insurer at any time would be the policyholder surplus, which is defined as the insurer’s admitted assets minus its liabilities, i.e., its net worth. In this model, we treat the profit accumulated in previous periods as the policyholder surplus, and thus we ignore the effect of investments and other lines of business. We assume the company starts its business at time \( y = 0 \) with a surplus (denoted as \( \mu^{s_0} \)) equal to \( k \) times the annual premiums \( Z_{imcv} \) (Eq. 6) received from homeowners of all building inventories in all risk regions, where \( k \) is a user-specified constant (Eq. 21).

\[
\mu^{s_0} = k \sum_{imcv} Z_{imcv}, \forall s
\]  

(21)

To determine the surplus \( \mu^{s,y} \) in each year, we make two alternative assumptions, and for each model run, we choose one (i.e., either Equation 22a or 22b is included in the formulation, but not both). In the capped surplus version (Eq. 22a), we assume the primary insurer reallocates surplus greater than this amount in each year \( y \) by reinvesting in other lines of business or distributing it to investors as dividends. The surplus in scenario \( s \) and year \( y \) then is the minimum of the sum of the profit in \( y \) and the surplus in \( y = 1 \), and the maximum allowable surplus \( k \sum_{imcv} Z_{imcv} \) (Eq. 22a).

\[
\mu^{s,y} = \min(\mu^{s,y-1} + F^{s,y}_k \sum_{imcv} Z_{imcv}) \forall s,y
\]  

(22a)

Our model allows for an alternative to reinsurance through a retained taxed surplus version in which the primary insurer might at least partially self-insure. In that case, we remove the cap on the surplus and instead assume the insurer will reinvest all after-tax surpluses within this business, where the tax rate is \( c \). As such, these investments are considered to be a reserve available to pay homeowners claims in extreme events, and the surplus in scenario \( s \) and year \( y \) is defined instead as in Equation 22b. This assumption relates to the argument in (Jaffee and Russell, 1997) that the failure of catastrophe insurance market lies in the inability of insurance companies to arrange for the level of capital necessary to settle extraordinarily large losses.

\[
\mu^{s,y} = \mu^{s,y-1} + (1 - \chi)F^{s,y} \forall s,y
\]  

(22b)

In both cases, if the accumulated surplus \( \mu^{s,y} \) in year \( y \) equals zero or less, we assume that the insurer becomes insolvent, and the profit \( F^{s,y} \) and
surplus $\mu^{sy}$ are set to zero for the remaining years ($y + 1, \ldots, Y$) of the scenario $s$.

**Insurer objective function.** The objective function is to maximize the average annual profit over the full time horizon, averaged over all scenarios $S$ (Expression 23). The model thus chooses values of the decision variables $A$ and $M$ defining the reinsurance treaty, subject to Constraints (14) to (22). This stochastic optimization model, which is both non-linear and non-convex, is solved using an automatic two-stage Response-Surface and Trust-Region solution procedure in Gao et al. (2014).

$$\text{Max } \frac{1}{SY} \sum_{sy} F^{sy}$$  \hspace{1cm} (23)

Note that this objective is equivalent to minimizing the total cost $C_j(\Sigma_o q_o)$ to the insurer, since total insurer cost equals the total premiums collected from the homeowners as income minus the profit, and the premium income is fixed at the specified price level. In other words, the optimal solution to Expression 23, $F^{sy*}$, will result in the optimal cost structure $C_j(\Sigma_o q_o)^* = C_j(\Sigma_o D_v(p_v))^*$ for the given price levels $p_v$ (Eq. 24).

$$C_j(\Sigma_o q_o)^* = \Sigma_o p_v q_v - F^{sy*}$$  \hspace{1cm} (24)

**CASE STUDY INPUTS**

We use eastern North Carolina as the study region. This area is subject tropical storms/hurricanes about every four years (SCONC, 2010). We focus on single-family wood-frame homes, the wind and storm surge flooding hazards (not rainfall-induced flooding), and direct losses (structural, non-structural, interior, mechanical, electrical, and plumbing, but no contents or additional living expenses). In order to understand the special variation in wind speeds and flood depths associated with a hurricane, we use the 2010 census tracts but subdivide each of the 143 census tracts that are located on the coast into up to three smaller zones: the area that is within one mile of the coastline, one to two miles from the coastline, and the remainder of the census tract. This yields 732 locations $i$.

Eight building categories $m$ were defined to represent all combinations of number of stories (one or two), garage (yes or no), and roof shape (hip or gable). Each building is defined as a collection of components represented in the damage and loss modeling (e.g., roof covering, openings). Each component in turn is made of many component units (e.g., a single window
or section of roof covering). For each component a few possible physical configurations are defined, each with an associated component resistance. The building resistance $c$ of each building is then defined by the vector of resistances of its components. The case study includes 192 building resistance levels (Peng, 2013).

The component-based loss simulation model is adapted from a modified Florida Public Hurricane Loss Model for the wind-related damage (FPHLM, 2005) and Taggart and van de Lindt (2009) and van de Lindt and Taggart (2009) for the flood-related damage. Described in detail in Peng et al. (2013) and Peng (2013), the loss model was used to compute the loss $L_{imcv}^h$ to a building in location $i$ of building type $m$ and resistance level $c$ in risk region $v$ given hurricane $h$. The building inventory data ($X_{imcv}$) was estimated using census data, with total building counts allocated among the building resistance levels $c$ based on location (coastal or not) and year built relative to major building code and construction practice changes.

We define two risk regions $v$—one region is all properties within two miles of the coast (high risk $H$) and the second region is all properties that are more than two miles from the coast (low risk $L$). The choice of risk aversion parameter to use as a benchmark is a challenge. The risk aversion parameter drives the optimizing choice to purchase and thus has an associated penetration rate. We have modeled a system in which the choice to purchase insurance is a voluntary expected utility maximization decision. Our model in its current form does not include current federal mandates or mortgage lender requirements so therefore represents a system not constrained by those requirements. We elected to use a benchmark penetration rate to back out an average coefficient of risk aversion. Evidence on insurance penetration rates for voluntary insurance indicates a wide range of possibilities.\(^4\) We elected to use average penetration rates for the National Flood Insurance Program reported in Dixon et al. (2006) to back out a mean risk aversion parameter value $\theta_v$ in the homeowner utility model (Gao, 2014). Specifically, values of $\theta_v$ were chosen so that given our utility model, they would result in average penetration rates consistent with those reported in Dixon et al. (2006). The parameter values are $\theta_H = 3.0 \times 10^{-5}$ and $\theta_L = 1.7246 \times 10^{-5}$ for relatively high and low risk areas, respectively. Extensions of the current paper will explore changes in the average coefficient of risk aversion and corresponding penetration rates and their effect on profitability and sustainability of the insurance firms and the overall performance of the integrated modeling system.

\(^4\)For example, King et al. (2014) report that the proportion of losses covered by insurance (related to penetration rate) for recent earthquake events ranged from 1% to 81%. 
We applied the Optimization-based Probabilistic Scenario method utilizing the set of 97 probabilistic hurricane scenarios $h$ developed in Apivatanagul et al. (2011). For each hurricane scenario, open terrain 3-second peak gust wind speeds and surge depths were computed throughout the study region using the storm surge and tidal model ADCIRC (Westerink et al., 2008). This set of hurricanes was shown to result in errors small enough to be inconsequential for regional loss estimation. Using those hurricanes, we developed a set of $S = 2000$ thirty-year scenarios that represent the full set of possible scenarios with minimal error (Peng, 2013). There are twenty time steps per year and $T = 600$ time steps per scenario $s$.

Other input parameter values include deductible $d = $5000, minimum premium required $p = $100, and homeowner insurance budgets of $\kappa_H = 5\%$ and $\kappa_L = 2.5\%$ of building value for high and low risk homeowners, respectively.\footnote{Although affordability of insurance premiums is the subject of a National Academy of Science study at this time and an issue of direct relevance to the Homeowner Flood Insurance Affordability Act of 2014, the scholarly literature does not suggest a definitive test for affordability, other than membership of the budget set given prices and income.} We also assumed primary insurer administrative loading factor $\tau = 0.35$ (personal communication, John Aquino, WillisRe), factor defining capped surplus $\kappa = 3$, tax rate for surplus retained $\chi = 0.4$, co-participation factor $\beta = 95\%$, reinsurer loading factor $\varphi = 0.1$, and two values of reinsurer risk attitude $\rho = 0.1$ and $0.5$, representing soft and hard reinsurance markets, respectively.

In summary, our case study region includes 1536 possible types of structures in a total inventory of 931,902 buildings. We estimated both wind and flood damage for each type of building at each location. The loss calculation includes 2000 scenarios and is embedded in the homeowner and insurance model with endogenously determined equilibrium price and inventory of insured structures. For convergence, each optimization requires at least 250 evaluations, and takes an average of 27 seconds to solve. We implemented all calculations in parallel on a Unix machine with 12 cores requiring approximately 12 hours to complete the solution.

**CASE STUDY RESULTS**

**Inverse Demand Functions and Cost Function**

We generate inverse demand functions and cost functions required for the case study application of the Cournot-Nash model. For each risk region $v$, we compute these functions for the range of price (per dollar of expected loss) levels from $1.35$ to $5.35$ with a step size of $0.001$ for the demand
function and $0.1 for the cost function. The value of $1.35 represents a zero profit price for the insurer that just covers the transaction costs for the insurer ($0.35 per dollar expected loss).

The approximated inverse demand curves are shown in Figure 3, where the low risk insurance demand is less sensitive to price than high risk. The demand in the low risk region is relatively inelastic. Much of this inelasticity is driven by the cut-off value for insurance. We implemented a minimum threshold total cost of the policy of $100 and assume is the insurer is unwilling to manage a policy that drops below the threshold value. This means that the expected loss plus markup must be at least $100 after a $5,000 deductible. As the markup increases and price rises, more homeowners in the low risk region exceed the $100 minimum threshold and become eligible to purchase insurance. As price increases, more low risk homeowners will optimally self-insure and a few of the most risk averse homeowners will bump up against the budget constraint that no more than 2.5% of the total value of the home may be devoted to an annual insurance premium. Relaxing these constraints will tend to increase the proportion of homes covered by insurance in the low risk region.

Through nonlinear regression, we create polynomial estimates of the inverse demand curves (Eq. 24), each with an adjusted $R^2 > 0.9$.

\[
P_H(q_{Hj}) = 2.945(10^{-33})q_{Hj}^4 - 2.59(10^{-24})q_{Hj}^3 + 8.406(10^{-16})q_{Hj}^2 - 1.249(10^{-7})q_{Hj} + 9.287 \tag{24a}
\]

\[
P_L(q_{Lj}) = 6.309(10^{-15})q_{Lj}^2 - 5.814(10^{-7})q_{Lj} + 14.68 \tag{24b}
\]

Evaluating the insurer cost $C_j(\Sigma v_{ij})$ at each of the 1600 price level combinations ($400$ for $p_H \times 400$ for $p_L$), we can plot the cost function as a
surface. We create cost functions for three separate cases: (1) capped surplus in a soft reinsurance market (i.e., using Eq. 22a, $g = 0.1$), (2) capped surplus in a hard reinsurance market (i.e., using Eq. 22a, $g = 0.5$), and (3) retain taxed surplus in a hard reinsurance market (i.e., using Eq. 22b, $g = 0.5$). Figure 4 shows the cost function for Case 1, with the capped surplus in a soft reinsurance market. Figure 4 suggests that the insurer’s total cost increases more quickly as its coverage in the high risk region expands in contrast to the low risk region. The cost functions for Cases 2 and 3 look similar, with vectors of coefficients $[2.272(10^7), 2.24, 2.533, -1.165 (10^{-6}), -8.03(10^{-6})]$ and $[1.966(10^{-7}), 2.197, 2.541, -1.087(10^{-6}), -7.459(10^{-6})]$, respectively (Gao, 2014). Similarly, we used a polynomial form to approximate the resultant cost surface, with an adjusted $R^2 > 0.9$ (Eq. 25).

\[
C(q_{Hj}, q_{Lj}) = 1.087(10^7) + 2.598q_{Hj} + 3.133q_{Lj} - 1.779(10^{-9})q_{Hj}^2 - 1.339(10^{-8})q_{Hj}q_{Lj}
\]

(25)

Note that the cross partial derivative of cost to insured loss in high and low risk area equals a negative constant $-1.339(10^{-8})$, which implies that there is some cost complementarity between the two distinct markets. This stems from the fact that a firm can spread its reinsurance cost across both
markets, which makes it more profitable to serve both markets as opposed to two separate firms (with separate cost functions) serving each market individually.

**Cournot-Nash Model Results**

With the fitted inverse demand functions and cost function, we apply the Cournot-Nash model (Eq. 1 to 5), by solving the first-order conditions (Eq. 4 and 5). Since the fitted inverse demand and cost functions are in polynomial form, the first-order conditions are a system of polynomial equations with two variables. We solve these equations using MATLAB version 7.12.0.635 (2012) for a specified number of insurers $n$. Since the highest-order polynomial is four, there are, at most, four solutions that satisfy the first-order conditions. From that set we select the solution that results in the highest profit per insurer, which dominates the other solutions. Finally, we denote the optimal coverage for a single insurer $j \in (1,2,...,n)$ as $(q^*_{Hj}, q^*_{Lj})$. We then obtain the optimal price per unit coverage for each region by substituting the coverage into the inverse demand functions: $p^*_{Hj} = P_H(nq^*_{Hj}), p^*_{Lj} = P_L(nq^*_{Lj})$.

We check the stability of the Cournot-Nash model by computing the norm of the partial derivative matrix (Eq. 5). More specifically, we calculate the one-norm of the partial derivative matrix for each solution for a given number of insurers. The results suggest that our Cournot-Nash solutions from monopoly to oligopoly (for $n = 1,2,...,10$) all satisfy the sufficiency condition for (Cournot-Nash equilibrium) stability, i.e., $\|\mathbf{T}(Q^*_{H}, Q^*_{L})\|_1 < 1$, where $Q^*_{H} = (q^*_{H1},...,q^*_{Hn})$, $Q^*_{L} = (q^*_{L1},...,q^*_{Ln})$.

The Cournot-Nash model results provide the key trends across these equilibrium solutions as the market structure changes from monopoly ($n = 1$) to less and less concentrated oligopoly $2 \leq n \leq 10$, (i.e., as insurer competition increases). We present the following as functions of the number of insurers in the market: (1) equilibrium price (per dollar of coverage) and insurance penetration (i.e., insured loss divided by total loss), (2) insurer’s performance in terms of profit, return on equity, and insolvency, and (3) reinsurance decisions. For each section, we present results for the three cases: (1) capped surplus in a soft reinsurance market, (2) capped surplus in a hard reinsurance market, and (3) retain taxed surplus in a hard reinsurance market.

**Equilibrium Price and Insurance Penetration**

Due to the cost complementarity between the distinct insurance markets/risk regions, an insurer could cross-subsidize its business in the low risk region with its business in the high risk region, which enables the insurer to offer coverage with a unit price lower than the fair price. It is
interesting to notice that the decline in price as the number of insurance carriers increases is less dramatic for the low risk region than the high risk region. For Case 1, for example, in the low risk area, the price decreases 25% from $1.20 to $0.90 as we move from one to ten insurers. In the high risk area, the same change is 42%, from $5.00 to $2.90.

As competition drives down the price in the high risk areas, the total insured loss and insurance penetration increase until the insurance penetration is 35% when there are ten primary insurers in the retain-hard market case (Fig. 5a). Comparing the three cases suggests that the insurance penetration is higher in the hard reinsurance market, and in the presence of policies that encourage establishment of larger reserves by the primary insurers (i.e., allowing them to retain profit, as in Case 3). This makes sense because the price is lower in those cases. In the low risk area, there are more modest increases in insurance penetration (Fig. 5b) as the number of insurers.

We adopted an empirical approach for estimating the cost for insurer(s) to operate in different risk regions, without any assumption on cross-subsidy between these regions. Under this approach, each insurer has the option of operating in both or just one of these risk regions. The (optimal) operational costs are estimated for all possible combinations (approximated by a finite set) of its business strategies and insurance volume determined by the individual homeowner choices. The approximated optimal cost surface implies that there is some cost complementarity between the two risk regions, and it is beneficial for the insurer to spread its risk into both risk regions (i.e., operate in both markets rather than serve each market individually). This implies the benefit of diversifying risk geographically for the insurer.

**Fig. 5.** Insurance penetration and total expected annual insured loss vs. number of insurers, for (a) high risk region and (b) low risk region, for the three cases.
Insurers grows because the impact of competition on pricing is more modest than in the high risk region. Note that the penetration is higher for the low risk area than the high risk area because the total loss is smaller in the former, and penetration is defined as insured divided by total loss. The low risk region homeowners can optimally opt out of the insurance market. Our approximated cost surface obtained from iterative optimization (based on internal risk model) implies that there exists efficient diversification and management of risk from both the risk regions.

**Insurer’s Performance**

We can also examine how each insurer’s performance changes with the number of insurers in the market. Here we measure an insurer’s performance using three metrics: (1) average annual profit, (2) average annual return on equity, and (3) average annual probability of insolvency. Average annual profit is as given in Equation 20. Return on equity (ROE) measures profitability by showing how efficiently capital is being used. Investors seek a high and stable return on equity, which is defined here as the annual net profit divided by the average surplus between the beginning and end of the year. Here we compute the average annual ROE, denoted as $R$, for the years $\eta$ that the insurer is solvent (Eq. 26). To calculate the annual probability of insolvency $\alpha$, we define $\phi^S$ to be a binary indicator variable that is one if the insurer becomes insolvent at any time in scenario $s$ and zero otherwise, and take the average over all scenarios $s$ (Eq. 27).

\[
R = \frac{1}{S} \sum_{\eta} \sum_{y=\eta} \frac{F^{s_y}}{0.5(\mu^{s_y-1} + \mu^{s_y})}
\]

\[
\alpha = \frac{1}{SY} \sum_{S} \phi^S
\]

Figure 6a shows how average annual return on equity (ROE) changes with changes in market concentration. Comparing the capped-hard market and retain-hard market cases (Cases 2 and 3) suggests that the ROE is higher for an insurer with a capped surplus than one retaining after-tax profits, because the surplus (and, therefore, ROE denominator) is smaller in the former case. This case also reflects the fact that the insurer is retaining more risk. As illustrated in Figure 6b, the annual probability of insolvency is substantially higher (more than 50%) with the capped surplus (Case 2) than without (Case 3), all else being equal. Comparing the soft and hard reinsurance markets with capped surplus (Cases 1 and 2) in Figure 8b suggests that the probability of insolvency is higher when it is more difficult to buy reinsurance (Case 2). Even when reinsurance is available
in the soft market (Case 1), when more insurers enter the market (especially at more than five insurers), it becomes more expensive relative to their profits, and thus insurers buy less of it and their probability of insolvency increases.

**Reinsurance Decisions**

As the number of insurers in the market changes, the use of reinsurance changes as well. To first understand how the reinsurance is used, we can compare the three cases. When the reinsurance market is soft, insurers are assumed not to keep an additional surplus of funds; hence they rely on reinsurance to avoid insolvency. By transferring risk, primary insurers obtain a better loss profile. Figure 7 illustrates the cumulative density function (CDF) of the net liability of a single primary insurer for the high risk region when the market includes five primary insurers, where net liability refers to the net loss the primary insurer is liable for after deductible and reinsurance payments are received. Among the three cases, the expenditures are lower in a soft reinsurance market. By contrast, more of the risk is retained by the primary insurer under a hard reinsurance market (with or without capped surplus) (Fig. 7). In the hard market case, reinsurance is not cost effective compared to retained surplus. Of course this benefit comes at a cost—the reinsurance expenses are substantially higher (Fig. 8). Figure 8 shows how the average annual expenditures for reinsurance, including base and reinstatement premiums, change with
An insurer’s dependence on reinsurance is dramatically reduced when reinsurance is more expensive (i.e., in a hard market) and/or under intense competitive pressures (i.e., when there are more insurers in a soft market.

Figure 9 shows the CDF of the reinsurer’s liability under different market configurations for the capped-soft reinsurance market. For Case 1,
although the annual average insurance payment per insurer declines as the number of insurers in the market increases, the total liability transferred to the reinsurer increases. As more insurers join the market, the loss/risk from the entire primary insurance market that is transferred to the reinsurer increases as well.

**SUMMARY**

In this study, we integrated a Cournot-Nash equilibrium framework with individual optimization models for key stakeholders (homeowners and insurers), and a loss estimation model. These tools were applied to a full-scale case study for hurricane risk (flood-wind combined) to residential buildings in eastern North Carolina. We examined the impact of competition in this voluntary catastrophe insurance market by investigating the market equilibrium price and insurance penetration; insurer’s performance in terms of profit, return on equity, and insolvency; and its decisions in mitigating financial risk through either reinsurance or self-insurance using cash reserves (retained surplus). This approach represents internal risk modeling that determines the adequacy of capital and financial risk management. As stated by Klein and Wang (2009), “the use of an internal risk model rather than a standard formula to determine its solvency capital requirement (SCR) is a significant forward step in insurer solvency regulation.” Insurance regulation in European Union countries
has moved towards requiring companies to perform internal risk modeling, whereas U.S. property-casualty insurers are not allowed to use internal risk modeling to demonstrate the adequacy of their capital and financial risk management. This paper contributes to the body of knowledge on internal risk modeling for risk management at all possible rate combinations in different risk regions.

The results from our study indicate that the level of concentration in the primary insurance market can lead to significant differences in the operational decisions for an individual insurance firm. Choices on reinsurance and retained surplus and therefore resultant insolvency rates can change dramatically with changes in the number of firms competing in the market. As expected, increasing the number of insurers, while creating challenges for the primary insurers, is attractive to the homeowners. Also, more risk is transferred to the reinsurers when there are more primary insurance carriers in the market. Hence, there is an important balance to be maintained between covering as many properties as possible in the region and maintaining the profitability/solvency of the carriers. That is the purpose of reserve requirements for insurance companies before they can offer insurance in a state which on is a barrier to entry—so limits competition—while reducing the likelihood of insolvency. Hence, models of this nature provide useful information for regulators to make public policy decisions on the balance between increased competition that is attractive to policyholders and insurer solvency rates as well as minimum cash reserve requirements.

There is empirical evidence to suggest that, with respect to low-probability but high-consequence loss events, homeowners tend to underestimate the low probabilities and therefore make non-optimal decisions by not explicitly considering the expected benefits and costs of different alternatives (Kahneman and Tversky, 2000; Kunreuther et al., 2004; Kunreuther 2006). This implies that there are limitations on the expected utility model we adopted in this study in capturing homeowner behavior. Therefore, treatment of homeowner demand for insurance that incorporates behavioral models of risk perceptions, limited information, and other forms of bounded rationality are important topics for future research.

REFERENCES


