Pricing Options with Physically Based Exercise and Random Maturity

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Abstract: Hurricane bonds are uniquely structured to comprise two exercise conditions: a physically based condition that the underlying hurricane makes landfall at a pre-specified location and a standard moneyness condition that they end in the money. As the time of landfall is uncertain, their maturities are also uniquely random. Thus extant loss-based catastrophe bond pricing models cannot be directly applied. We propose a novel solution at the nexus of physical science and finance by integrating hurricane risk modeling and option pricing modeling to produce a coupled and physically based hurricane bond pricing model. We apply this methodology to pricing parametric hurricane bonds, featuring a parametric trigger based on physical parameters related to hurricane tracks, radius, and intensity, as they have become increasingly popular in hurricane risk management. As hurricane arrival is underpinned by the discrete, sporadic, and random arrival of hurricane-specific information, we implement hurricane risk modeling by Monte Carlo simulation in information time. Our physically based model has the potential advantage of being able to apply to direct hedges against hurricane intensity and size uncertainties at landfall. [Key words: physically based exercise structure; hurricane risk modeling; option pricing modeling.] JEL classification: G13; G17; G22.

INTRODUCTION

Scientific evidence[4] has shown that the increase in greenhouse gas concentrations since the Industrial Revolution has caused global warming with rising sea surface temperatures, inducing increasing arrival of extreme weather events. To mitigate this risk exposure on capital

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markets, insurance-linked securities (hereafter ILS) have evolved from being a threatened reinsurance substitute in the 1990s to becoming a viable complementary reinsurance product. This development underpins the convergence of the ILS and traditional reinsurance markets. According to a report by Swiss Re (2018),[5] such alternative ILS capital accounted for 22% of total property catastrophe reinsurance limit in 2017, growing to around $95 billion in the first half of 2018, four times larger than it was in 2010. A recent Artemis (2018)[6] article further indicates that this ILS capacity will grow to about $100 billion by the end of 2018, following the substantial catastrophe losses in 2017 from Hurricanes Harvey, Irma, Jose, and Maria, and rampant California wildfires. Among the three main types of ILS—catastrophe bonds (cat bonds), industry loss warranties (ILW), and collateralized reinsurance—cat bonds are the only instruments that are truly securitized for which a secondary market exists, and according to an Artemis article (October 4, 2016, entitled "Cat Bonds in Holding Pattern; Florida on Watch for Hurricane Matthew") around 60% of the outstanding catastrophe bond market remains exposed to U.S. hurricane risks. These so-called "hurricane bonds" are unique in that they have a dual exercise structure such that they are only exercisable should the underlying hurricane make landfall at a preset physical location.

Among hurricane bonds of different trigger types, the parametric/parametric index triggers have been gaining popularity in the catastrophe space in recent years due to transparency and rapid payout after an event,

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[4]Among the evidence provided is a comprehensive idealized hurricane intensity modeling study by Knutson and Tuleya (2004), which uses future climate projections from nine different global climate models and four different versions of the GFDL (Geophysical Fluid Dynamics Lab) hurricane model. According to this study, an 80-year buildup of atmospheric CO_2 at 1%/yr with compounding leads to roughly a one-half category increase in potential hurricane intensity on the Saffir-Simpson scale and an 18% increase in precipitation near the hurricane core. A 1%/yr CO_2 increase is an idealized scenario of future climate forcing. An implication of the study is that if the frequency of tropical cyclones remains the same over the coming century, a greenhouse-gas induced warming may lead to an increasing risk in the occurrence of highly destructive category-5 storms. Webster, Holland, Curry, and Chang (2005), Emanuel (2005) and others have all provided empirical evidence that global warming has been increasing the severity and destructive power of hurricanes over the past few decades. Emanuel (2006) further showed that a 10% increase in water temperature would increase hurricane intensity by 65%, whereas a 10% increase in shear would decrease hurricane intensity only by 12%.

[5]http://www.artemis.bm/news/ils-to-grow-continue-to-dampen-reinsurance-market-cycle-swiss-re/, entitled "ILS to grow, continue to dampen reinsurance market cycle: Swiss Re," December 4, 2018.

[6]https://www.artemis.bm/news/nephila-capital-adds-11-to-reach-12-2bn-of-ils-assets-managed/, entitled "Nephila Capital adds 11 % to reach $12.2bn of ILS assets managed," July 17, 2018.

vis-à-vis other triggering designs such as indemnity and industry-loss-indexed triggers in which cases claims' calculations may be opaque and payouts may take several months. Parametric insurance features a parametric trigger, which is typically based on parameters directly related to the risk that the insured seeks to acquire coverage against, such as hurricane wind speed, hurricane minimum central pressure, temperature, rainfall total, geographic location of a storm, etc. Its coverage is gaining traction as hazard modeling continues to improve, weather stations more accurately gauge wind speeds, and satellites capture images that reveal the extent of flood damage. Payment occurs when defined triggering parameters are met, offering bespoke protection with the certainty of timing and rapid hassle-free payment. The payment itself is also an agreed-upon figure prior to the event. Thus, there is no need for claims adjustment after the event occurs. It also has the advantage of being able to apply directly to hedge against hurricane intensity and size uncertainties in hurricane risk management. Multinational insurer AXA Corporate describes its parametric department as one that "uses new types of information and big data processing methods to build custom-made insurance covers against unexpected weather events." [7]

Unfortunately, despite the need to incorporate the landfall exercise feature and to subsume the important role physically based parametric triggers play in parametric insurance, extant catastrophe (cat hereafter) bond pricing models are not set up to cope with these issues. There are two distinct approaches to price catastrophe bonds: the empirical actuarial science approach and the theoretical mathematical finance approach. In the former, catastrophe bonds are empirically priced according to the following pricing norm: Layer premium = Expected layer loss + Risk load factor x Standard Deviation, where the expected layer loss and standard deviation are obtained from complex physically based industry catastrophe models inclusive of the consideration of the modeled factors of input quality, model errors, and output uncertainty, while the risk load factor is a subjective entry that depends on risk preference, capital position, and market conditions, among other considerations. Actuaries also load non-modeled market factors on top of the modeled factors to increase the premium. For example, Braun (2016) employs a primary-market dataset to analyze cat bond spreads using the actuary approach and finds in a linear regression analysis that, apart from the expected loss, the covered territory, the sponsor's rating, the reinsurance cycle, and the spreads on comparably rated corporate bonds all exhibit a significant impact. One drawback of this empirical approach is that it does not stem from any finance theory—e.g.,

[7] https://www.insurancejournal.com/news/national/2017/11/22/472010.htm.

option pricing theory—and thus it cannot deal with the dual exercise issue. Thus in this paper we focus on developing an integrated mathematical finance approach across hurricane risk modeling and option pricing modeling.

In mathematical finance, cat bonds with indemnity/industry loss index triggers have been priced as if they were traded in perfect financial markets by using option pricing theory, non-arbitrage martingale pricing theory, and utility optimization theory. In lieu of incorporating the loss outputs from an industry cat modeler, these models assume cat loss arrival processes are exogenously given as a compound Poisson process or its variations, as in Bowers, Gerber, Hickman, Jones, and Nesbitt (1986); as such they are loss- but not physically based. More recently, Jarrow (2010) suggests risk loading the Poisson arrival parameter in order to calibrate risk premium and drive a closed-form cat bond pricing formula; Härdle and Cabrera (2010) apply cat bond data on the May 2006 Mexican earthquake to calibrate a pure parametric cat bond and argue that under a homogenous Poisson arrival setting the implied arrival frequency rate can measure the relative costs of reinsurance and cat bond allocations; Gatzert, Pokutta, and Vogl (2019) resolve the non-tradability of the underlying insurance loss processes by forming (approximate) replicating portfolios for ILW positions from index-based cat bonds; Chang, Yang, and Yu (2018) incorporated climate and CO_2 indices to improve modeling of hurricane arrival frequency. Although these models are well-calibrated, none of them are set up to cope with the dual exercise/random maturity issue as well as to subsume parametric triggers.

Our contributions to the literature are three-fold: (1) We propose a novel interdisciplinary solution at the nexus of finance and physical science by integrating hurricane risk modeling and option pricing modeling to produce a coupled and physically based option pricing model. To our best knowledge, this coupling approach is the first of its kind in the literature. (2) To demonstrate this methodology, we price hurricane bonds with a parametric trigger, while current catastrophe bond models are all loss-based. (3) This physically based model has the potential advantage of being able to apply to direct delta-gamma hedges against hurricane intensity and size uncertainties at landfall. In hurricane risk management, risk managers are concerned about hedging the uncertainties of hurricane intensity and size at landfall. Taking Hurricane Michael (2018) as an example for intensity uncertainty, the NHC (National Hurricane Center) issued a five-day warning forecast showing it would make landfall near Mexico Beach, Florida, with a wind intensity of 80 mph. It actually landed five days later at Mexico Beach but with a quick shift to a wind intensity of 155 mph, one of the four most powerful storms ever to make landfall in the US. When it came ashore, meteorologists said, it was like a 20-mile-wide tornado. Let us take Hurricane Florence (2018) as an example for size uncertainty; it landed in New Jersey as a much wider hurricane than anticipated at initiation, with very severe flooding. Since, unlike other cat bond pricing models that are based on loss estimates, our coupled model is physically based, it thus has the potential advantage of being able to apply to direct delta-vega hedges against hurricane intensity and size uncertainties at landfall.

In this research our hurricane index is the two-factor CME Hurricane Index (CHI),[8] a numerical physical measure composed of hurricane intensity and radius metrics to calibrate the potential for damage from a hurricane. As hurricane arrival is underpinned by the discrete, sporadic, and random arrival of hurricane-specific information (e.g., Wu and Chung, 2010), we implement the coupled model in information time.[9] In hurricane risk modeling, we assume that the evolution of hurricane intensity and radius (the parent process) follow a geometric Brownian motion model with the seminal empirical hurricane track model of Vickery, Skerlj, and Twisdale (2000) being the subordinator, i.e., the directing process, to simulate hurricane synthetic tracks. The Vickery, Skerlj, and Twisdale model

[8]Chang, Chang, and Wen (2014) have used the CHI to derive optimum hurricane futures hedge in a warming environment.

[9]The concept of "randomized operational time," which originated in probability theory (Feller, 1971), is widely applied in systems and engineering fields. It actuates subordinating process (SP hereafter) modeling by randomizing the time scale of a stationary parent process using a new operational time scale called the subordinator or the directing process. The choice of this operational scale is dictated by the nature of things. In the variability of security returns, empirical studies (e.g., Ané and Geman, 2000) have validated the model prediction that this variability relates positively to the number of transactions during a given calendar time period so a time change from calendar time to transaction time will allow the leptokurtosis of return distribution to be significantly reduced. Chang, Chang, and Lim (1998; CCL hereafter) applied this concept to transform a calendar-time call option with stochastic volatility to an isomorphic information-time call option with random maturity (to reflect the randomness of information arrivals), resulting in a parsimonious call option pricing formula as a risk-neutral Poisson sum of Merton's (1973) prices over the option's information-time maturity domain with only two unobservable variables—the information arrival intensity and the per-arrival underlying volatility. In the variability of insurance claims, Chang, Chang, and Yu (1996) subordinated futures price changes to transactions on cat futures contract and implemented a stochastic time change from calendar time to transaction time. They then derived a transaction-time option pricing formula as a risk-neutral Poisson sum of Black's (1976) prices over the option's transaction-time maturity domain. As Geman and Yor (1997) have suggested, unlike other pricing models, the Chang, Chang, and Yu model is unique in having the merit of illuminating the information conveyed by transactions. Chang, Chang, and Lu (2010) further apply a doubly-binomial SP framework to price an Asian-style cat option in claim time with non-traded underlying loss index and mean-reverting Ornstein-Uhlenback claim intensity process.

has later been extended by Yonekura and Hall (2011) and others and is widely used in the cat modeling industry (see Emanuel, 2017) with the understanding that natural disaster prediction is a typical data-driving scientific application at the nexus of machine learning, statistical modeling, and database technology. This track simulation allows us to check if and when a hurricane will make landfall at a preset location, while the evolution of intensity and radius along hurricane tracks dictates the index value evolution leading to the final exercise resolution. Given these physically based outputs, we next couple it to the information-time option pricing model of Chang, Chang, and Lim (1998; CCL hereafter)[10] to price a hurricane bond.

HURRICANE RISK MODELING

Hurricane risk assessment makes use of Monte Carlo simulation to generate synthetic basin-based storms to obtain landfall statistics; two principal approaches have been developed. The leading industry risk models are based on large sets of synthetic storms generated from the statistics of historic tracks with intensity evolutions based on observed intensities and their relationships to environmental predictors such as sea surface temperatures, a method pioneered by Vickery et al. (2000) and extended by Yonekura and Hall (2011) and others. The starting position, date, time, heading, and translation speed of all tropical storms are given in the HURDAT database of the National Hurricane Center. Using the historical starting position, date, and time of the storms ensures that the climatology associated with any seasonal preferences for the point of storm initiation is retained. Given the initial storm heading, speed, and intensity, the simulation model estimates the new position and speed of the storm based on the changes in the translation speed and storm heading over the current 6-h period. The prime advantage of this empirical modeling approach compared to the traditional approaches is the ability to properly model hurricane wind risk over large regions, within which the hurricane

[10]We choose CCL as our information-time option pricing model as Chang, Chang, and Yu (1996) and Chang, Chang, and Lu (2010) apply only to Asian-style cat options. It is noted that CCL is a risk-neutral pricing scheme with the assumption of a traded underlying asset, but the CHI is not traded. To resolve this issue, Jarrow (2010) suggests exploiting information embedded in prices of traded cat bonds as a way to risk load the Poisson arrival parameter that drives trigger events. Recently, Gatzert et al. (2019) resolve the non-tradability of the underlying insurance loss processes by forming (approximate) replicating portfolios for ILW positions from index-based cat bonds.

climatology can vary significantly. This is useful for assessing the risk of large-scale systems, such as for addressing insurance issues, for which policies may be distributed along the entire coastline. The drawback is that it relies entirely on historical data and thus cannot readily incorporate or predict future changes due to, for example, global warming. This is especially true for tropical cyclones (TC hereafter, and we use the terms tropical cyclones, hurricanes, and typhoons interchangeably) due to their limited historical records.

As a functional alternative to the empirical track models, attempts have been made in recent years in the natural science community to simulate TC by using physically based models driven by the large-scale climate conditions provided by global reanalysis data and/or by global climate models. Emanuel, Ravela, Vivant, and Risi (2006) generated synthetic hurricane tracks by seeding weak protovortices randomly in space and time based on historical genesis locations, which then move with synthesized winds generated using global reanalysis data incorporating the earth's sphericity and rotation, a method also termed a "beta-and-advection model" (Marks, 1992). This model can then be coupled with the ocean-atmosphere TC model (CHIPS; Emanuel, DesAutels, Holloway, and Korty, 2004) to simulate intensities along each track. Emanuel, Sundararajan, and Williams (2008) developed a synthetic event generator in which storms are generated by random seeding of large-scale climate states rather than by using historical data, and then use the CHIPS intensity model to determine which seeds survive. Because such a method is based purely on physics applied to large-scale environmental conditions, it can be driven by climate model output as well as by global reanalysis data and is thus more flexible than the Emanuel et al. (2006) model. Emanuel (2017, and FAST hereafter) further presented a faster intensity simulator that can produce large numbers of TC events to assess the impact of climate change on TC intensity with much shorter computer run time. The FAST model performance was shown to be comparable to that of the CHIPS model in a select number of hindcasts of 2004 hurricanes.

As a novel attempt to develop a coupled model across finance and physical science, in this paper we simulate hurricane tracks using the relatively simple Vickery et al. (2000) model, as the implementation of the physically based FAST model is much more challenging, requiring the choice of a global climate model to couple with the CHIP intensity model. Obviously we do expect that the implementation of a more advanced model such as FAST would improve track simulation, leading to better pricing. For the simulation of the CHI values in our subordinated process framework, which requires predictions of radius R and velocity V, we assume that the evolution of R and V along hurricane tracks follows a geometric Brownian motion model. Given R and V, one can make a deterministic prediction of the CHI value.

PRICING A PARAMETRIC HURRICANE BOND IN AN OPTION-THEORETICAL FRAMEWORK

Two types of parametric hurricane bonds exist. For a per-occurrence one to be triggered, the indexed loss value from a single natural disaster must exceed the attachment point; in contrast, an annual-aggregate one references all losses from covered perils that occur within one calendar year (stop-loss reinsurance). Here we price a per-occurrence hurricane bond with a parametric trigger in an option-theoretical framework. This bond has random maturity as the landfall time is uncertain and stipulates a dual exercise structure whereby the underlying hurricane must make landfall at a pre-set location and the underlying hurricane index value end in-the-money. Conditional upon meeting this physically based exercise condition, the cat bond's payoff at the hurricane's landfall, denoted as P_T, can be commonly specified as follows:

$$P_T = \begin{cases} F & \text{if } I_T < K, \\ F - \delta_T & \text{if } I_T \geq K, \end{cases} \tag{1}$$

where I_T is the value of the index at the landfall, T the landfall time, and F the face value of the cat bond that its holders will receive when I_T does not reach the predetermined trigger level K. For notational convenience we assume the tick size is \$1 per index point, though in practice the tick size is set according to the face value of the bond; and δ_T represents the total amount that will be forgiven by cat bondholders when the trigger level has been pulled, in which case bondholders will receive only $F - \delta_T$. We further assume

$$\delta_T = \begin{cases} I_T - K & \text{if } K \leq I_T < K + F, \\ F & \text{if } I_T \geq K + F. \end{cases} \tag{2}$$

The cat bond value can thus be quickly shown to be

$$P_T = F - Max[0, I_T - K] + Max[0, I_T - (K + F)], \tag{3}$$

corresponding to the payoffs of a portfolio of three components: a long position in a riskless zero-coupon bond, a short position in an index call with strike price K, and a long position in an index call with strike price $K + F$, i.e., a long bond plus a short bull spread with calls (see Loubergé et al. (1999) for this observation).

To price the call option with a physically based exercise condition we expand CCL to couple option pricing modeling to hurricane risk modeling. This extension to incorporate hurricane risk modeling is necessary as for a hurricane option to be exercisable, the underlying hurricane track must hit a preset landfall area. That is, we would need to model not only the index value evolution but also to the hurricane tracks.

Conditional upon the hurricane making landfall at a preset location, CCL dictates that the call value can be formulated as a risk-neutral Poisson sum of the index option prices over the option's information-time maturity domain:

$$C = \sum_{k=0}^{\infty} \Gamma(k,jT) C_M(I,X,B_T,\sigma_1,k), \tag{4}$$

where $\Gamma(k,jT) = e^{-jT}\frac{(jT)^k}{k!}$ is a Poisson distribution with information arrival intensity parameter j and $C_M(I,X,B_T,\sigma_1,k) = IN(d_1) - B_T XN(d_2)$ is the Merton (1973) call option pricing formula with stochastic interest rates such that

$$d_1 = \frac{ln(I/B_T X) + \left(\frac{1}{2}\sigma_1^2\right)k}{\sigma_1\sqrt{k}} \text{ and } d_2 = d_1 - \sigma_1\sqrt{k}, \tag{5}$$

where I, T, X, B_T, and σ_1 denote the index value, the time of landfall, the matching bond price, the strike price, and the per-arrival index volatility, respectively. Since we price a per-occurrence but not a fixed-maturity hurricane bond, we do not have to include coupon payments. Next we will show how to couple CCL to a hurricane track model.

APPLICATION TO A PARAMETRIC TRIGGER USING THE CME HURRICANE INDEX (CHI)

Instead of being based on any claims (the insurer's actual claims, the modeled claims, or the industry's claims), a parametric trigger is indexed to natural hazard parameters such as the wind speed (for a hurricane bond), the ground acceleration (for an earthquake bond), or another metric appropriate for the peril, with data for these parameters collected at multiple reporting stations and then entered into specified formulae. For example, if a typhoon generates wind speeds greater than X meters per second at 50 of the 150 weather observation stations of the Japanese Meteorological Agency, the cat bond is triggered. Another example is the Bosphorus 2015-1 earthquake cat bond issued by the Turkish Catastrophe Insurance Pool. For an earthquake to qualify as a triggering event and cause a loss, it must result in spectral acceleration (a gravity-based measure of ground movement and shaking from earthquakes) above 0.1g for at least 10% of the defined measurement locations affecting the Istanbul region of Turkey and be confirmed by the calculation agent.

The CME Hurricane Index (CHI) that we consider is a numerical physical measure of the potential for damage from a hurricane. It is modeled after Carvill's Hurricane Damage Potential Index (HDPI), as shown below, published in April 2006 and maintained and calculated by EqeCAT. CHI is a continuous measurement starting from zero and has no maximum value:

$$CHI = \left(\frac{V}{Vo}\right)^3 + \frac{3}{2}\left(\frac{R}{Ro}\right)\left(\frac{V}{Vo}\right)^2, \tag{6}$$

where R is the radius of the extent of hurricane-force winds from the center of the storm, i.e., "size"; V is the velocity, i.e., the maximum sustained wind speed in miles per hour, or "intensity"; and the subscript 0 denotes reference values. For example, on the CME, the reference values of 74 mph and 60 miles were used for the maximum sustained wind speed and radius of hurricane-force winds, respectively. The higher the CHI value the more destructive the hurricane. The commonly used Saffir-Simpson Hurricane Scale (SSHS) classifies hurricanes in categories from 1 to 5 based on only velocities but not radius and thus cannot properly measure the actual physical impact, making it less than optimal for use by the insurance/investment community and the public at large. As a case in point, Katrina was described as a weak category 4 storm at the time of its landfall but this did not provide a real estimate to the actual physical impact, while Hurricane Wilma in 2005 was at one point in its life the strongest storm on record. The CHI highlights that, at its strongest, Hurricane Katrina was a far wider storm and had more potential for damage than Wilma, despite its lower wind speed. The SHS would have been unable to make this distinction clear.

In the following, we simulate an Atlantic Seaboard hurricane or a Gulf States hurricane.

Synthetic Storm Track Simulation

The first step of the track simulation is to produce the storm genesis locations in the Atlantic Ocean. Storm genesis locations are uniformly sampled within a box that has a latitude range of 10–20 N and longitude range of 60–0 W. We assume an initial hurricane translation speed of 1.307 m/s (or approximately 0.03 lat/long per hour). For the hurricane heading, we compare two scenarios by assuming a north (or 0 degrees) heading and a northwest (or 45 degrees) heading. Given a genesis location, the storm latitude and longitude are iteratively updated every time step of six hours till either the storm makes landfall at the boundary of the Atlantic Seaboard or the Gulf States. As a common practice, we approximate the landmass of the Atlantic Seaboard as having a boundary with the Atlantic Ocean at a longitude of 75 W and latitude range of 30–42 N. The Gulf States have a boundary at a latitude of 30 N and longitude range of 75–100 W.

In Vickery et al. (2000), the storm heading and the natural logarithm of the storm translation speed are linear functions of the heading and speed at the previous time step as well as storm latitude and longitude. A Gaussian random error term is added to the heading and log-speed predictions for stochasticity. The changes in the translation speed c and storm heading e between times i and $i + 1$ are obtained from

$$\Delta \ln c = a1 + a2k + a3v + a4 \ln c_i + a5 \mathrm{e}_i + \varepsilon \tag{7a}$$

$$\Delta \mathrm{e} = b1 + b2\kappa + b3v + b4c_i + b5\mathrm{e}_i + b6\mathrm{e}_{i-1} + u \tag{7b}$$

where $a1$, $a2$, etc. = constants; κ and ν = storm latitude and longitude, respectively; c_i = storm translation speed at time step i; e_i = storm heading at time step i; e_{i-1} = heading of the storm at time step $i - 1$; and ε = random error term. A different set of coefficients for easterly and westerly headed storms is used. As the simulated storm moves into a different 5° X 5° square, the coefficients used to define the changes in heading and speed change accordingly. In the case of grid squares with little or no historical data (e.g., East Atlantic Ocean and squares near the equator), the coefficients in (1) for these squares are assigned the values for those determined for the nearest grid square.

Here in information time, we randomize the number of information arrivals according to a Poisson distribution with monthly intensity j of 100 and will only update the storm location (latitude and longitude) for each track when hurricane-specific information arrives in a time step. We define an information-time-step as a time step over which information arrives. As our purpose is to demonstrate a theoretical modeling methodology rather than a bona fide empirical study, we assume for simplicity constant coefficients that $a1 = 2.5e - 4$, $a2 = 5e–5$, $a3 = 5e–5$, $a4 = 5e–5$; $a5 = 5e–5$, $b1 = 2.5e–3$, $b2 = 5e–4$, $b3 = 5e–4$, $b4 = 5e–4$, $b5 = 5e–4$, and $b6 = 5e–4$, and that the error terms follow normal distribution $N(\mu, \sigma)$ as $N(0, 5e–5)$ in equation (7a) and $N(0, 5)$ in (7b) to arrive at the typical values in Vickery et al. (2000). We simulated 5000 synthetic tracks in information time, among which 461 of the tracks (9.22%) made landfall at the preset area if the initial heading is north or 0 degrees (Figure 1a). When the initial heading is northwest or 45 degrees, 1674 of the 5000 tracks (33.48%) made landfall at the preset area (Figure 1b). For each track making landfall that triggers the option exercise, we also record the landfall arrival time as the first passage time, T. The landfall time distribution is displayed in Figures 2a and 2b. Synthetic tracks not making landfall are displayed with a landfall time of 0. For the track making landfall, we find the average landfall time to be 35.39 days and 32.48 days when the initial heading is respectively north and northwest.

CHI Simulation and Option Pricing

To produce predictions of the CHI value at every information-time-step, one needs predictions of the radius R and velocity V. The evolution of R and V is modeled using a geometric Brownian motion model, with per-information-arrival drifts of 0.0001 and 0.0001, respectively, and volatilities of 0.02 and 0.02, respectively. The CME respective reference values for R and V of 60 miles and 74 mph are used as the initial values. Given R and V, one can make a deterministic prediction of the CHI. Figure 3 displays the simulated paths of R and V and the resulting evolutions of the CHI. Figure 4 displays the ending CHI value distributions.

Based on the number of information arrivals in the sample paths and the volatility of the unconditional CHI value distribution, the per-arrival index volatility $\sigma 1$ can be calculated and used as inputs to the CCL call value pricing formula, e.g., equation (4). To make a preliminary observation on the usefulness of the information-time formula in pricing the hurricane index call spread and the cat bond contract it embeds in, we obtain numerical values of equation (4) for the short call (with strike K), the long call (with strike $K + F$), and the cat bond (long bond with short call spread), respectively. We assume that the predetermined trigger level K is 10, the face value of the cat bond F is 8, and the present value operator or the price of a riskless matching bond B_T is 0.99. Additionally, we assume that the current CHI levels I are 2.5, 10, 18, and 20, respectively, to represent deep-out-of-the-money, at-the-money for short call, at-the-money for long call, and in-the-money, respectively. We assume a monthly information arrival intensity parameter j to range from 50 to 500, and the per-arrival index volatility σ_1 to range from 1% to 10%. Finally, the probability of landfall of

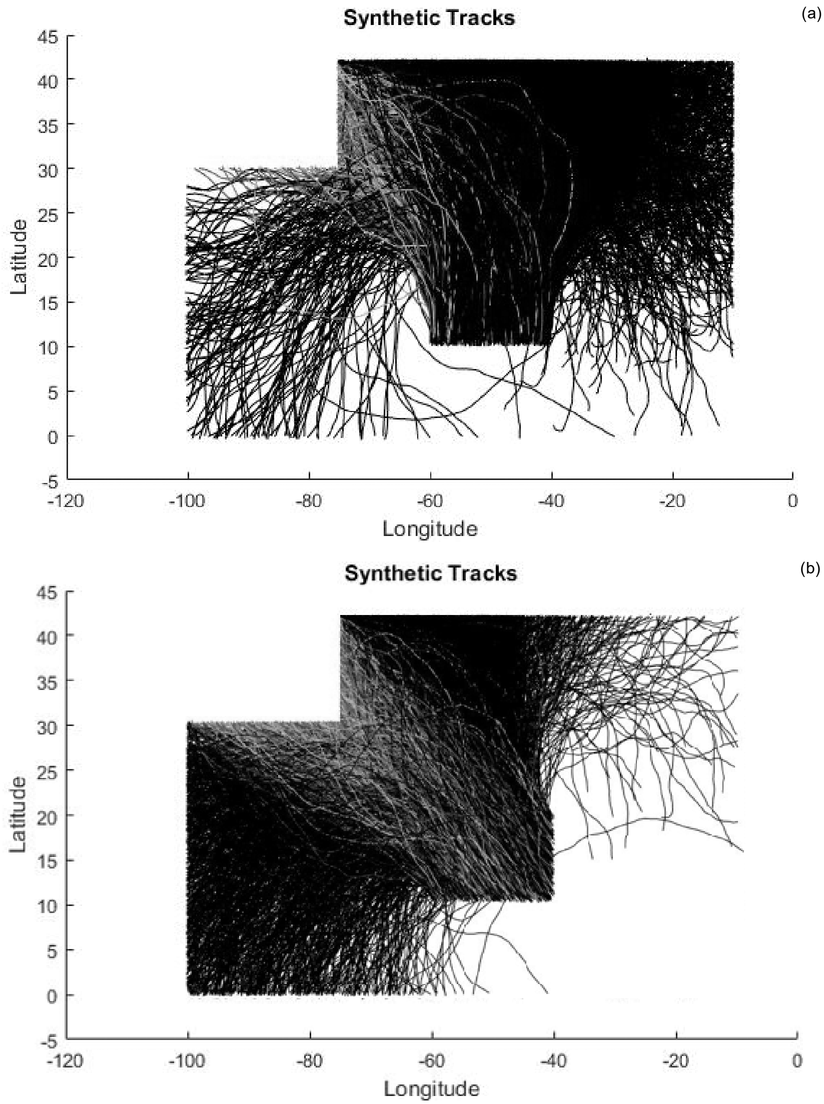

Fig. 1. Synthetic tracks of hurricane eye. Storm genesis locations are uniformly sampled within a box of 10–20 N and 60–40 W. The latitude and longitude are iteratively updated per information arrival until the storm makes landfall at a longitude of 75 W and latitude range of 30–42 N, or at latitude of 30 N and longitude range of 75–100 W. We assume an initial heading of 0 degrees (Figure 1a) or 45 degrees (Figure 1b). Lighter-black colored lines are synthetic tracks making landfall at the preset area.

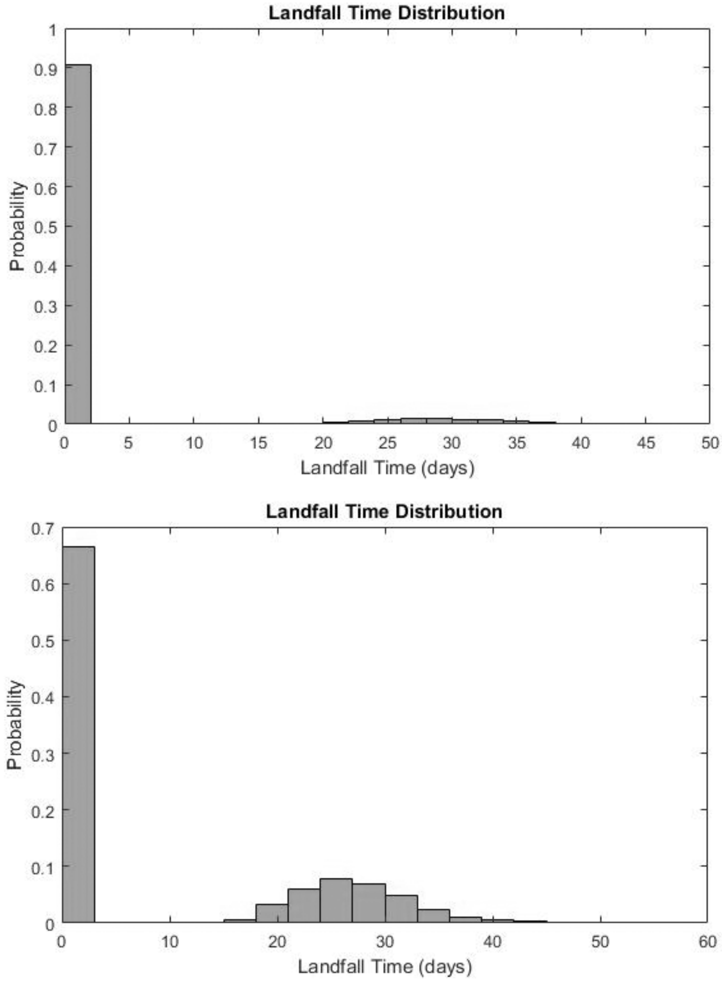

Fig. 2. The distribution of landfall arrival time. The distribution of landfall arrival time in days given the initial heading of 0 degrees (Figure 2a) and 45 degrees (Figure 2b). Synthetic tracks not making landfall are displayed with a landfall time of 0.

33.48% (1674 out of 5000 simulated tracks) is used to calculate the unconditional value of call options; i.e., it is the conditional call value from the CCL formula multiplied by a factor of 0.3348.

Table 1 Panel A reports the values of short call options V_{short}, Panel B reports the values of long call options V_{long}, while Panel C reports the cat

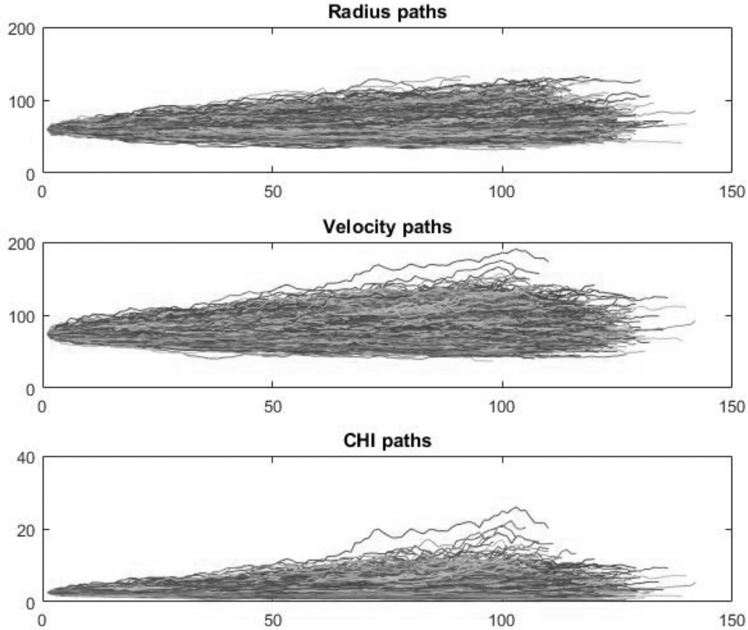

Fig. 3. The simulated paths of radius, velocity, and the resulting evolutions of the CHI. The evolution of radius and velocity is modeled using a geometric Brownian motion model, with per-information-arrival drifts of 0.0001 and 0.0001, respectively, and volatilities of 0.02 and 0.02, respectively. The CME respective reference values for R and V of 60 miles and 74 mph are used as the initial values.

bond values. As predicted by the formula, the call values are positively associated with the CHI index level and the index volatility σ_1, but are negatively associated with moneyness. In addition, the higher the information arrival intensity parameter j, the higher the call values. Finally, the cat bond values are negatively related to the call spread values.

CONCLUDING REMARKS

Although up to 60% of the outstanding catastrophe bond market remains exposed to the U.S. hurricane risks, extant loss-based catastrophe bond models are not set up to price hurricane bonds, as these bonds' exercises are based on natural hazard metrics; e.g., the bond is only exercisable should the underlying hurricane make landfall at a preset location. We make a novel attempt to integrate hurricane risk modeling and option pricing modeling to produce a coupled and physically based

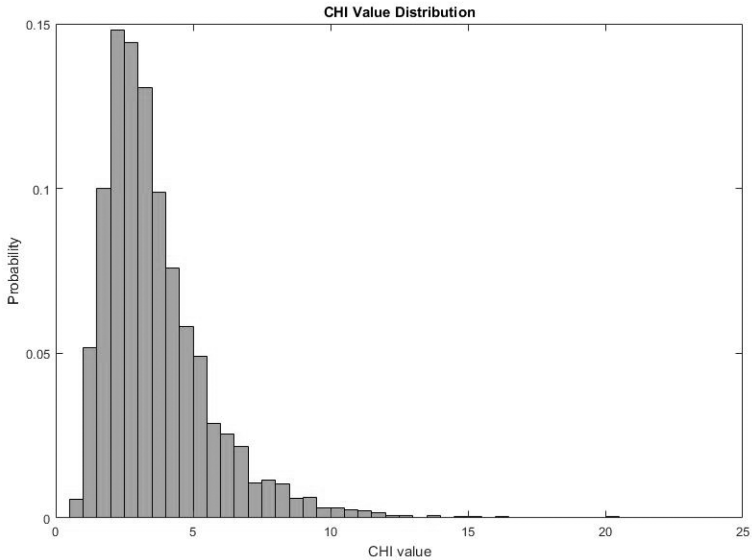

Fig. 4. The distribution of CHI ending values in the simulated paths. The evolution of R and V is modeled using a geometric Brownian motion model, with per-information-arrival drifts of 0.0001 and 0.0001, respectively, and volatilities of 0.02 and 0.02, respectively. Given R and V, one can make a deterministic prediction of the CHI. The initial value of CHI is 2.5.

option pricing model to resolve this issue. Specifically, we implement a simplified version of the empirical hurricane track model of Vickery, Skerlj, and Twisdale (2000) in information time to simulate hurricane tracks, and apply a geometric Brownian motion model to simulate hurricane intensity and radius evolution along these tracks. By coupling this simulation approach with the information-time option pricing model of Chang, Chang, and Lim (1998), we price a hurricane bond indexed to a hurricane's destructive power built upon hurricane intensity and radius. This coupling methodology implies the potential of similar applications to other physically based options such as earthquake options with geoscience modeling of earthquake location and earthquake-induced spectral acceleration force, as well as longevity options with bio-science modeling of longevity growth.

Finally, in hurricane risk management, risk managers are concerned about hedging the uncertainties of hurricane intensity and size at landfall. Taking Hurricane Michael (2018) as an example for intensity uncertainty, the NHC (National Hurricane Center) issued a five-day warning forecast showing it would make landfall near Mexico Beach, Florida, with a wind intensity of 80 mph. It actually landed five days later at Mexico Beach but

Table 1. Option and Cat Bond Pricing

The predetermined trigger level K is 10, the face value of the cat bond F is 8, and the present value operator B_T is 0.99. The current CHI levels I are 2.5, 10, 18, and 20, respectively, to represent deep-out-of-the-money, at-the-money, in-the-money, and deep-in-the-money cases. The information arrival intensity parameter j ranges from 50 to 500, and the per-arrival index volatility σ_1 ranges from 1% to 10%. Call option values are first calculated according to equation (4) and then adjusted by multiplying the probability of landfall. The values of short call options V_{short} are displayed in panel A. The values of long call options V_{long} are displayed in panel B. The values of cat bond $V = F^*B_T - V_{short} + V_{long}$ are displayed in panel C.

Panel A. Short Call Option (X = K) Values

		Intensity j			
	σ_1	50	100	200	500
I = 2.5	1%	0.00	0.00	0.00	0.00
	2%	0.00	0.00	0.00	0.00
	5%	0.00	0.00	0.01	0.09
	10%	0.01	0.06	0.18	0.44
I = 10	1%	0.11	0.15	0.20	0.31
	2%	0.20	0.28	0.39	0.61
	5%	0.48	0.67	0.94	1.43
	10%	0.93	1.29	1.75	2.47
I = 18	1%	2.71	2.71	2.71	2.71
	2%	2.71	2.71	2.72	2.79
	5%	2.74	2.84	3.05	3.58
	10%	3.05	3.42	3.96	4.87
I = 20	1%	3.38	3.38	3.38	3.38
	2%	3.38	3.38	3.38	3.43
	5%	3.40	3.47	3.65	4.16
	10%	3.65	4.00	4.55	5.49

Table continues

Panel B. Long Call Option (X = K + F) Values

	σ_1	Intensity j 50	100	200	500
I = 2.5	1%	0.00	0.00	0.00	0.00
	2%	0.00	0.00	0.00	0.00
	5%	0.00	0.00	0.00	0.04
	10%	0.00	0.02	0.10	0.36
I =10	1%	0.00	0.00	0.00	0.00
	2%	0.00	0.00	0.01	0.09
	5%	0.03	0.13	0.35	0.89
	10%	0.35	0.73	1.28	2.19
I = 18	1%	0.20	0.27	0.37	0.56
	2%	0.37	0.51	0.70	1.09
	5%	0.87	1.21	1.69	2.57
	10%	1.68	2.32	3.15	4.44
I = 20	1%	0.74	0.77	0.83	1.00
	2%	0.83	0.95	1.13	1.52
	5%	1.30	1.65	2.14	3.06
	10%	2.13	2.80	3.67	5.03

Panel C. Cat Bond Values

	σ_1	Intensity j 50	100	200	500
I = 2.5	1%	7.92	7.92	7.92	7.92
	2%	7.92	7.92	7.92	7.92
	5%	7.92	7.92	7.91	7.87
	10%	7.91	7.88	7.84	7.84
I = 10	1%	7.81	7.77	7.72	7.61
	2%	7.72	7.64	7.54	7.40
	5%	7.47	7.38	7.34	7.38
	10%	7.34	7.36	7.45	7.64
I = 18	1%	5.41	5.48	5.58	5.77
	2%	5.58	5.72	5.90	6.22
	5%	6.05	6.30	6.56	6.92
	10%	6.55	6.82	7.11	7.49
I = 20	1%	5.28	5.31	5.37	5.54
	2%	5.37	5.49	5.67	6.01
	5%	5.82	6.10	6.40	6.82
	10%	6.40	6.72	7.04	7.46

with a quick shift to a wind intensity of 155 mph, one of the four most powerful storms ever to make landfall in the US. When it came ashore, meteorologists said, it was like a 20-mile-wide tornado. Hurricane Florence (2018), as an example for size uncertainty, landed in New Jersey as a much wider hurricane than anticipated at initiation, with very severe flooding. Unlike other cat bond pricing models that are based on loss estimates, our coupled model is physically based and thus has the potential advantage of being able to apply to direct delta-gamma hedges against hurricane intensity and size uncertainties at landfall.

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